Homework #5: Due Friday October 16, 2015

1. Do Problem #5-10 in Ayyub & McCuen.

2. The annual maximum rainstorm produces rainfall (R) in inches at a site of interest that follow an extreme value type I (Gumbel) distribution with a mean of 20 in. and a standard deviation of 8 in.
   (a) Compute the magnitude of the 10-year storm (ie. 90th percentile).
   (b) The drainage system at the site can handle 25 in. of rainfall. Anything beyond that will result in flooding. Determine the probability that the drainage system will not meet demand for such events.

3. If two loads are applied to a cantilever beam as shown in the figure below, the bending moment at 0 due to the loads is $a_1X_1 + a_2X_2$.

   \[
   \begin{array}{c}
   X_1 \\
   \downarrow \\
   0 \\
   \end{array} \quad \begin{array}{c}
   X_2 \\
   \downarrow \\
   a_1 \\
   a_2 \\
   \end{array}
   \]

   (a) Suppose that $X_1$ and $X_2$ are independent random variables with means of 2 and 4 kips, respectively, and standard deviations of 0.5 and 1.0 kip, respectively. If $a_1 = 5$ ft and $a_2 = 10$ ft, what is the expected bending moment and what is the standard deviation of the bending moment?

   (b) If $X_1$ and $X_2$ are normally distributed, what is the probability that the bending moment will exceed 75 kip-ft?

4. Three different roads feed into a particular freeway entrance. Suppose that during a fixed time period, the number of cars coming from each road onto the freeway is a random variable, with expected value and standard deviation as given in the table below.

<table>
<thead>
<tr>
<th>Expected Value</th>
<th>Road 1</th>
<th>Road 2</th>
<th>Road 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation</td>
<td>16</td>
<td>25</td>
<td>18</td>
</tr>
</tbody>
</table>
(a) What is the expected total number of cars entering the freeway at this point during the specified time period?  [Hint: Let Xi represent the number of cars from road i.]

(b) What is the variance of the total number of cars entering the freeway?  Have you made any assumptions about the relationship between the number of cars entering from different roads?

5. Two water pumps are used at one of the pumping stations that supplies a large city.  Let X be the lifetime of the first pump and Y be the lifetime of the second pump.  The units of both X and Y are in thousands of hours, and X and Y are assumed independent.  Further, suppose X and Y are each normally distributed with:

\[ E[X] = 10, \text{Var}[X] = 1 \]

\[ E[Y] = 7.5, \text{Var}[Y] = 0.9^2 \]

Consider the difference in the lifetime of the two pumps: X – Y.

(a) Calculate the expected value and variance of the difference in the lifetime of the two pumps:  E[X – Y] and Var[X – Y].

(b) Determine the probability that the difference in the lifetime of the two pumps is less than 4,000 hours, i.e. P[X – Y ≤ 4].

6. The ductile strength (X) of a material is typically assumed to be lognormally distributed.  Suppose \( \mu_Y = 5 \) and \( \sigma_Y = 0.1 \).

(a) Compute the mean and variance of the ductile strength.

(b) Compute P(X > 125)

(c) Compute P(110 ≤ X ≤ 125).

(d) What is the value of the median ductile strength?

(e) If the smallest 5% of values were unacceptable, what would the minimum acceptable ductile strength be?
7. Suppose the ductile strength (X) of a material has a Gumbel distribution with $\alpha = 0.1$ and $\varepsilon = 110$.

(a) Compute the mean and variance of the ductile strength.

(b) Compute $P(X > 125)$

(c) What is the value of the median ductile strength?

(d) If the smallest 5% of values were unacceptable, what would the minimum acceptable ductile strength be?

8. Suppose the mean and variance of a random variable are

$$\mu = 6.267 \quad \text{and} \quad \sigma^2 = 10.73$$

Several continuous probability distributions have been covered this semester, including

(i) Normal

(ii) Lognormal

(iii) Gumbel

(iv) Weibull

Answer questions (a) and (b) below assuming that the random variable has each of these distributions. For the Weibull distribution, assume the shape parameter $k$ has a value of 2.

(a) What are the parameters of the four distributions?

(b) What are the 1, 50, and 99 percentiles of the four distributions?