CE 3710: Uncertainty Analysis in Engineering

**Homework #3:** Due Friday September 25, 2015

1. Do the following problems in Ayyub & McCuen:

   #3-32  **For problem #3-32, also compute P(X ≤ 4) and P(3 < X ≤ 5).**
   
   #3-35
   
   #3-48

2. Consider a Little League team that has 15 players on its roster.

   (a) Without considering the actual position to be played, how many ways are there to select 9 players for the starting lineup?

   (b) Without considering the actual position to be played, how many ways are there to select 9 players for the starting lineup and a batting order for the 9 starters?

   (c) Suppose 5 of the 15 players are left-handed. How many ways are there to select 3 left-handed outfielders and have the 6 other positions occupied by right-handed players?

3. Shortly after being put into service, some buses manufactured by a certain company have developed cracks on the underside of the main frame. Suppose a particular city has 25 of these buses, and cracks have actually appeared in 8 of them.

   (a) How many ways are there to select a sample of 5 buses from the 25 for a thorough inspection?

   (b) In how many ways can a sample of 5 buses contain exactly 4 with visible cracks?

4. A box in a certain supply closet contains four 40-W light bulbs, five 60-W bulbs, and six 75-W bulbs. Suppose that three bulbs are randomly selected.

   (a) What is the probability that exactly two of the selected bulbs are rated 75 W?

   (b) What is the probability that all three of the selected bulbs have the same rating?

   (c) What is the probability that one bulb of each type is selected?
5. Consider the possible probability mass functions (pmf) provided in the table below.

<table>
<thead>
<tr>
<th>x_i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_1(x)</td>
<td>0.3</td>
<td>0.4</td>
<td>0.1</td>
<td>0.05</td>
<td>0.05</td>
<td>0.0</td>
</tr>
<tr>
<td>P_2(x)</td>
<td>−0.2</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>P_3(x)</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(a) Which of the possible pmfs are legitimate? Briefly indicate why.

(b) For the legitimate pmf, compute the mean (μ) and variance (σ^2).

6. “Time headway” in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X represent the time headway (seconds) for two randomly chosen consecutive cars on a freeway during a period of heavy flow. Suppose the following pdf represents distribution of time headway (X) in a given traffic environment:

\[ f_X(x) = \begin{cases} \frac{k}{x^4} & x \geq 1 \\ 0 & x < 1 \end{cases} \]

(a) Determine the value of k for which \( f_X(x) \) is a legitimate pdf.

(b) Obtain the cumulative distribution function.

(c) Use \( F_X(x) \) from part (b) to determine the probability that headway exceeds 2 seconds.

(d) Determine the probability that headway is between 2 and 3 seconds.

(e) Compute the mean and variance of the time headway.

(f) Compute the median and the 90th percentile of the distribution.
7. The reading given by a thermometer calibrated in ice water (actual temperature 0°C) is a random variable with probability density function

\[ f_X(x) = \begin{cases} 
\frac{3}{4}(1-x^2) & -1 < x < 1 \\
0 & \text{otherwise}
\end{cases} \]

(a) What is the probability that the thermometer reads above 0°C?

(b) What is the probability that the reading is within -0.25°C and +0.25°C?

(c) What is the mean reading?

(d) What is the standard deviation?

(e) Now consider the conversion of temperatures from °C to °F. If \( X \) = temperature in °C, and \( Y \) = temperature in °F, then the conversion is: \( Y = 1.8 \times X + 32 \). Given the mean and standard deviation of the readings in °C from parts (c) and (d), compute the mean and standard deviation of the readings in °F (\( \mu_Y \) and \( \sigma_Y \)).

NOTE: EXCEL has functions: COMBIN(n,k); FACT(m) which may be useful for some problems. Most calculators have similar functions as well.