Note Packet #5

Counting

CE 3710

September 11, 2015
Counting

• Method to determine number of outcomes contained in sample space (or some subset of sample space).

• Applies to cases with finite sample space.

• Basis for determining probabilities when all simple events are equally likely.

\[
P(A) = \frac{\text{# sample points in } A}{\text{# sample points in } S}
\]

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<td>Product Rule (Multiplication Principle)</td>
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Product Rule – Ordered sets WITH replacement

If sets $A_1, A_2, ..., A_k$ have respectively $n_1, n_2, ..., n_k$ elements, then the number of possible ways to select an element from $A_1$, then from $A_2$, ..., and finally from $A_k$ is

$$N = n_1 \times n_2 \times \ldots \times n_k$$

Example:

Suppose you need to form a committee of three composed of one sophomore, one junior, and one senior.

How many ways can you form the committee given 10 sophomores, 70 juniors, and 20 seniors to choose from?
**Product Rule** – Ordered sets WITH replacement

Example: Reconsider the network example from Lecture #4.
   How many possible outcomes are there (e.g. A up, B down, C up) given that each of the three components can be either up or down?

Example:
   Suppose $n = 5$ people share an apartment. On a given day, there are $k = 3$ jobs to do: shop, cook, and clean.
   If each person can do any number of jobs, how many ways are there to assign the jobs?
**Permutations** – Ordered sets without replacement

**Like product rule except each individual can do at most one job**

\[ N = n(n - 1)(n - 2)(n - 3) \cdots (n - k + 1) = \frac{n!}{(n-k)!} \]

**Example:**
Suppose 5 people share an apartment. On a given day, there are 3 jobs to do: shop, cook, and clean.
How many ways are there to assign jobs if each person can do at most one job?
**Combinations** – Unordered sets with OUT replacement

**Example:** From a group of $n$ people, $k$ must be chosen to serve on a committee. How many ways can a committee of 3 be formed out of 5 roommates?

The number of possibilities is:

$$\binom{n}{k} = \frac{n(n-1)(n-2) \cdots (n-k+1)}{k!} = \frac{n!}{k!(n-k)!}$$
Consider buses within the public transportation system of NYC. Of 100 buses, 20 have trouble with wheel chair lifts. If 10 buses are randomly selected and their lifts inspected, compute the following probabilities:

\[ P[ \text{none of 10 buses have trouble} ] = \]

\[ P[ \text{exactly 2 of 10 buses have trouble} ] = \]