Lecture 2: Elements of Probability

CE 3710

September 2, 2015
Probability Theory: Ayyub & McCuen § 3.2-3.3.2

- Introduce elementary properties of probability
- Discuss notation and definitions

Terms and Ideas

Probability

How we describe the chance (likelihood) of different events occurring

*Engineers must consider what events might occur as well as their relative likelihood and consequences.*
Experiment -- any process whose outcome is subject to uncertainty

Sample Space $S$ -- set of all possible outcomes
  (values in the population)

Sample point $x$ -- one single outcome

Event $E$ -- a subset of $S$

Simple Event -- a set containing a single sample point

Compound Event -- a set containing more than one sample point
Example: *The number of rainy days in one week*

Experiment (process): Measure daily rainfall

Sample space, $S$: What are all of the possible numbers of days it might rain in one week?

<table>
<thead>
<tr>
<th>SUN</th>
<th>MON</th>
<th>TUES</th>
<th>WED</th>
<th>THURS</th>
<th>FRI</th>
<th>SAT</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="rain.png" alt="Rain" /></td>
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</table>

**ANSWER:** 0, 1, 2, 3, 4, 5, 6, or 7 days of rain.
Example: *The number of rainy days in one week*

Sample point, x: What is a single outcome of this process?

**ANSWER:** Weather report for one week:

<table>
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<tr>
<th>SUN</th>
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<th>THURS</th>
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<tbody>
<tr>
<td><img src="image" alt="Sun" /></td>
<td><img src="image" alt="Cloud" /></td>
<td><img src="image" alt="Rain" /></td>
<td><img src="image" alt="Rain" /></td>
<td><img src="image" alt="Sun" /></td>
<td><img src="image" alt="Sun" /></td>
<td><img src="image" alt="Cloud" /></td>
</tr>
</tbody>
</table>
Example: *The amount of rain that fell in one week*

Experiment (process): Measure daily (weekly) rainfall

Sample space, $S$: What are all of the possible values of rainfall that may fall in one week?

![Rainfall Measurements](image)

**Answer:** The total amount of rain in one week can be any non-negative value (from 0 to infinity).

**Note:** The solution is a continuum of values that cannot be enumerated, versus in the last example, where only 8 values were possible.
Example: The amount of rain that fell in one week

Sample point, x: What is a single outcome of this process?

ANSWER: Total amount of rainfall in one week

In this week, it rained 3 inches.
Set Operations:

Let A and B be two events in the sample space S

• Union (U) – set of outcomes contained in A OR B OR Both

• Intersection (∩) -- set of outcomes contained in A AND B

• Complement (′ or c) – set of outcomes in S NOT in A

• Mutually exclusive or disjoint events; empty set ∅
Set Operations: EXAMPLE

Consider experiment where 100 cylinders are tested. A cylinder fails if not strong enough to meet standard.

Sample points: \( x = \{ \text{exactly } n \text{ cylinders fail} \} \)
where \( n = 0, 1, 2, \ldots 100 \)

Compound events:

\[ A = \{ \text{no more than 10 cylinders fail} \} = \{ 0 \leq n \leq 10 \} \]
\[ B = \{ \text{more than 5 cylinders fail} \} = \{ 6 \leq n \leq 100 \} \]
**Probability Function**

P( ) is a function that describes the probability of observing a given sample point or event contained in S.

A probability must be assigned to every outcome in S.

*We will discuss how to assign probabilities in Lecture #5: Counting.*

*For now…*

When all outcomes are equally likely, we can think of the probability of a particular event occurring as the fraction of time the event occurs relative to all possible outcomes in S.

**Example:** Rolling a die once.

There is a 1/6 chance that you roll a 1.

P(x = 1) = 1/6
Properties of Probability

1) For every event A, \(0 \leq P(A) \leq 1\)

2) If E cannot happen, then \(P(E) = 0\)

3) \(P(\emptyset) = 0\) where \(\emptyset\) is the empty set

4) For sample space S, \(P(S) = 1\)

5) For mutually exclusive events A and B, \(P(A \cap B) = 0\)
Properties of Probability

6) For any two events, \( P( A \cup B) = P(A) + P(B) - P(A \cap B) \)

\( \rightarrow \) Draw Venn Diagram

7) For every event \( A \), \( P(A) = 1 - P(A') \)

8) If \( \{ A_1, A_2, \ldots, A_n \} \) is a collection of mutually exclusive events, then \( P( A_1 \cup A_2 \cup \ldots \cup A_n ) = \sum P(A_i) \)
<table>
<thead>
<tr>
<th>Rule Type</th>
<th>Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identity rules</td>
<td>( A \cup \emptyset = A, \ A \cap \emptyset = \emptyset, \ A \cup S = S, \ A \cap S = A )</td>
</tr>
<tr>
<td>Idempotent rules</td>
<td>( A \cup A = A, \ A \cap A = A )</td>
</tr>
<tr>
<td>Complement rules</td>
<td>( A \cup \overline{A} = S, \ A \cap \overline{A} = \emptyset, \ \overline{A} = A, \ \overline{S} = \emptyset, \ \overline{\emptyset} = S )</td>
</tr>
<tr>
<td>Commutative rules</td>
<td>( A \cup B = B \cup A, \ A \cap B = B \cap A )</td>
</tr>
<tr>
<td>Associative rules</td>
<td>( (A \cup B) \cup C = A \cup (B \cup C), \ (A \cap B) \cap C = A \cap (B \cap C) )</td>
</tr>
<tr>
<td>Distributive rules</td>
<td>( (A \cup B) \cap C = (A \cap C) \cup (B \cap C) )</td>
</tr>
<tr>
<td></td>
<td>( (A \cap B) \cup C = (A \cap C) \cup (B \cap C) )</td>
</tr>
<tr>
<td>de Morgan's rule</td>
<td>( \overline{(A \cup B)} = \overline{A} \cap \overline{B}, \ (E_1 \cup E_2 \cup \ldots \cup E_n) = \overline{E}_1 \cap \overline{E}_2 \cap \ldots \cap \overline{E}_n )</td>
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<tr>
<td>Combinations of rules</td>
<td>( \overline{(A \cup (B \cap C))} = \overline{A} \cap (B \cap C) = (\overline{A} \cap \overline{B}) \cup (\overline{A} \cap \overline{C}) )</td>
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</tbody>
</table>
**Example**

Suppose you are trying to paint a building using scaffolding. You need to consider the weather (rain, wind conditions).

<table>
<thead>
<tr>
<th>Events (Probabilities)</th>
<th>No Rain (N)</th>
<th>Rain (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Wind (H)</td>
<td>HN (0.1)</td>
<td>HR (0.08)</td>
</tr>
<tr>
<td>Medium Wind (M)</td>
<td>MN (0.3)</td>
<td>MR (0.05)</td>
</tr>
<tr>
<td>Low Wind (L)</td>
<td>LN (0.4)</td>
<td>LR (0.07)</td>
</tr>
</tbody>
</table>

*Define two events:*

\[
A = \{ \text{no rain} \} = \{ \text{HN, MN, LN} \}
\]

\[
B = \{ \text{can use scaffolding} \} = \{ \text{M or L wind} \} = \{ \text{MN, MR, LN, LR} \}
\]