Note Packet #14

Method of Moments (MOM)

CE 3710
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Method of Moments

Overview of Procedure:
Natural estimator of the population mean $\mu$ is 

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$E[\bar{X}] = E\left[ \frac{1}{n} \sum_{i=1}^{n} X_i \right] = \sum_{i=1}^{n} E[X_i] / n = \sum_{i=1}^{n} \mu / n = \mu$$

$$\text{Var}[\bar{X}] = \text{Var}\left[ \frac{1}{n} \sum_{i=1}^{n} X_i \right] = \left( \frac{1}{n^2} \right) \sum_{i=1}^{n} \text{Var}[X_i] = \sigma^2 / n$$
A natural estimator of the variance $\sigma^2$ is

$$S^2 = \sum_{i=1}^{n} [X_i - \bar{X}]^2 / (n - 1)$$

$E[S^2] = \sigma^2$

$\text{Var}[S^2] = ??$
Example

Given a sample of independent observations \( \{x_1, \ldots, x_n\} \), estimate the corresponding distribution parameters using MOM assuming a Gumbel model.
Example

Let \{x_1, \ldots, x_n\} be a random sample of independent observations from a distribution with density function:

\[ f_X(x) = 0.564 a^{1.5} x^{0.5} \exp(-ax) \quad \text{for} \quad x \geq 0; \quad 0 \text{ otherwise} \]

For which the mean is: \( E[X] = 1.5/a \)

What is the method of moments estimator of \( a \)?
Example

An engineer believes that a bounded random variable $X$ has the following cumulative distribution function:

$$F_X(x) = x^\lambda \quad \text{for} \quad 0 < x < 1; \quad 0 \quad \text{for} \quad x \leq 1; \quad 1 \quad \text{for} \quad x \geq 1$$

for $0 < \lambda$. Given a random sample of $n$ independent observations $\{x_1, \ldots, x_n\}$, what is the method of moments estimator of $\lambda$?
Example

Given a sample of independent observations \( \{x_1, \ldots, x_n\} \), estimate the corresponding distribution parameters using MOM assuming the data follow a Normal distribution.
Example

Given a sample of independent observations \( \{x_1, \ldots, x_n\} \), estimate the corresponding distribution parameters using MOM assuming the data follow a lognormal distribution.