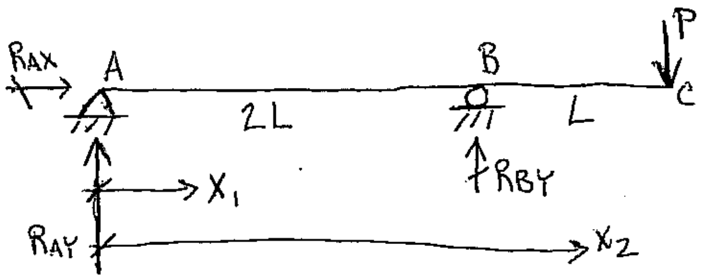
SOLUTIONS

PROBLEM 1.) DOUBLE INTEGRATION METHOD

a.) FOR THE x_1, x_2 COORDINATE SYSTEM SHOWN ON THE BEAM BELOW, FIND THE SLOPE AND DEFLECTION EQUATIONS. EI IS CONSTANT. EXPRESSIONS FOR MOMENT (M_1 & M_2) ARE GIVEN. (15 POINTS)



$0 \le x_1 \le 2L$

$$M_1(x_1) = -\frac{P}{2}x_1$$

$$EI\frac{dy_1}{dx} = -\frac{P}{4}x_1^2 + C_1$$

$$EI\,y_1 = -\frac{P}{12}x_1^3 + C_1x_1 + C_2$$

$2L \le x_2 \le 3L$

$$M_2(x_2) = Px_2 - PL$$

$$EI\frac{dy_2}{dx_2} = \frac{P}{2}x_2^2 - PLx_2 + C_3$$

$$EI\,y_2 = \frac{P}{6}x_2^3 - \frac{PL}{2}x_2^2 + C_3x_2 + C_4$$

b.) INDICATE 4 APPROPRIATE BOUNDRY CONDITIONS YOU *MIGHT* USE TO SOLVE FOR YOUR CONSTANTS OF INTEGRATION FROM PART a.). SET UP THE SOLUTIONS FOR YOUR CONSTANTS BASED ON THE BOUNDRY CONDITIONS YOU HAVE SELECTED BUT DO NOT SOLVE FOR THE CONSTANTS. (15 POINTS)

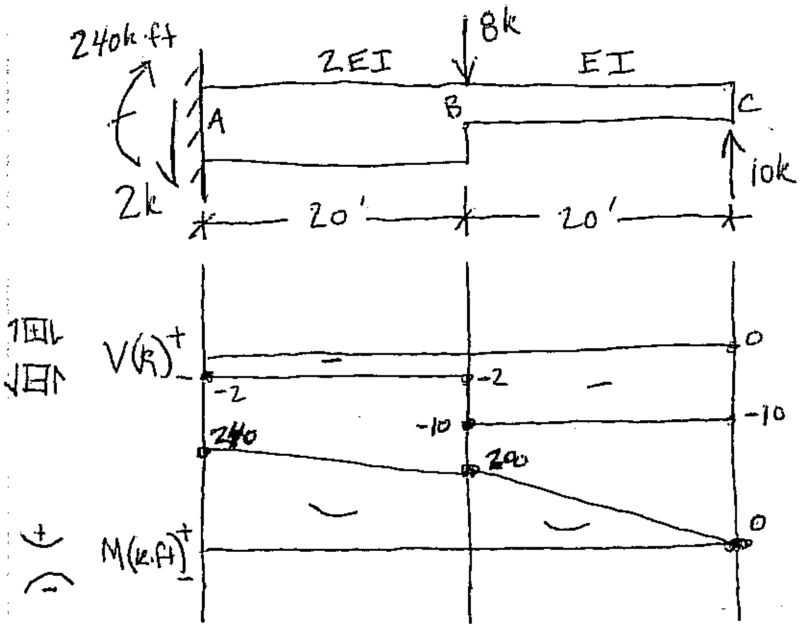
1) $y_1(0) = 0$; $EI(0) = -\frac{P}{12}(0)^3 + C_1(0) + C_2$

2) $y_1(2L) = 0$; $EI(0) = -\frac{P}{12}(2L)^3 + C_1(2L) + C_2$

3) $y_2(2L) = 0$; $EI(0) = \frac{P}{6}(2L)^3 - \frac{PL}{2}(2L)^2 + C_3(2L) + C_4$

4) $\frac{dy_1}{dx_1}(2L) = \frac{dy_2}{dx_2}(2L)$; $-\frac{P}{4}(2L)^2 + C_1 = \frac{P}{2}(2L)^2 - PL(2L) + C_3$

PROBLEM 2) CONSIDER THE FOLLOWING BEAM WITH CROSS-SECTIONAL PROPERTIES AND SHEAR & MOMENT DIAGRAMS SHOWN.



a.) FIND THE SLOPE AT B. USE THE MOMENT-AREA METHOD. (10 POINTS)

b.) FIND THE DISPLACEMENT AT C. USE THE MOMENT-AREA METHOD. (15 POINTS)

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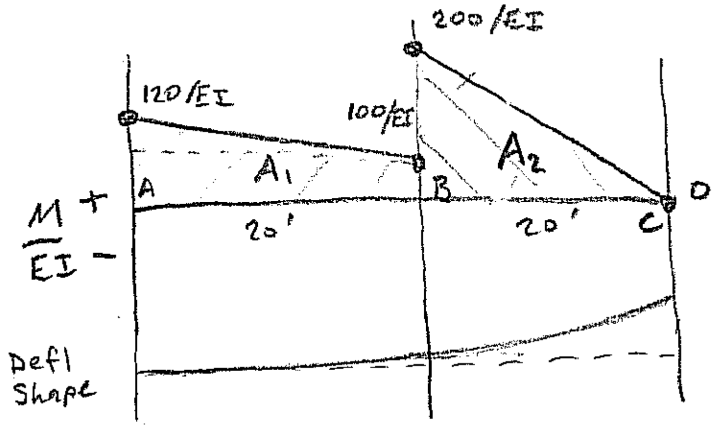


a.) Slope at B $= 0 + A_1$

$$= 0 + \left[\frac{1}{2}\left(\frac{20}{EI}\right)(20) + \left(\frac{100}{EI}\right)(20)\right]$$

$$\boxed{\theta_B = \frac{2200}{EI}} \ (k \cdot ft^2)$$

b.) $\Delta_C = t_{CA} =$ Moment A_1 + Moment A_2

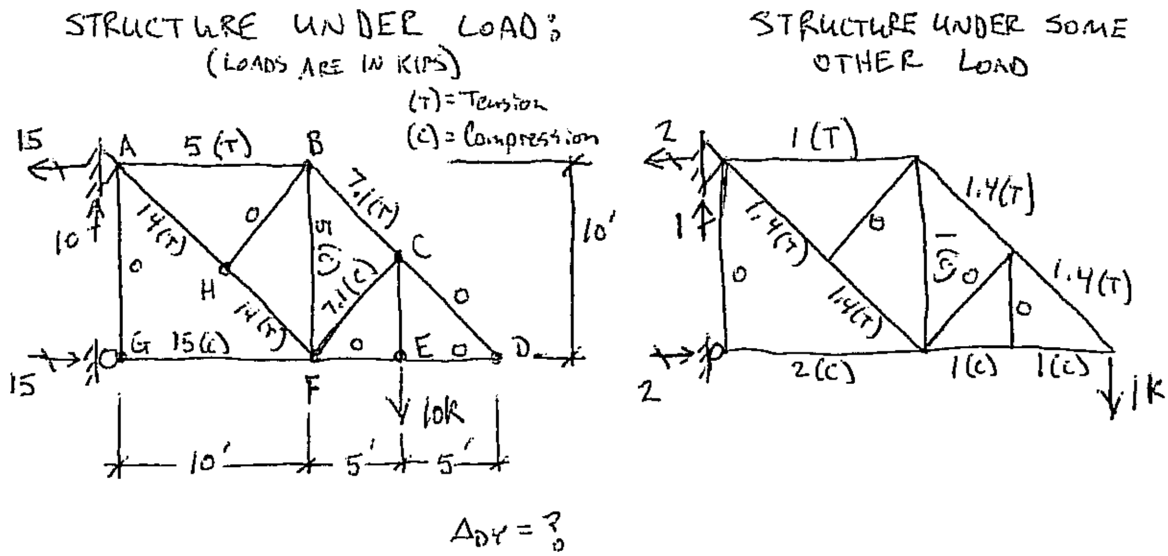
$$= \left(\frac{1}{2}\left(\frac{20}{EI}\right)(20)\left(\frac{2}{3}(20)+20\right) + \frac{100}{EI}(20)(20+10) + \frac{1}{2}\left(\frac{200}{EI}\right)(20)\left(\frac{2}{3}(20)\right)\right)$$

$$\Delta_C = \left(\frac{6667}{EI} + \frac{60{,}000}{EI} + \frac{26{,}667}{EI}\right)$$

$$\boxed{\Delta_C = \frac{93{,}300}{EI}} \ (k \cdot ft^3)$$

PROBLEM 3.) METHOD OF VIRTUAL WORK

a.) FOR THE TRUSS SHOWN BELOW, FIND THE VERTICAL DISPLACEMENT AT NODE D. FOR ALL MEMBERS, E=29,000ksi, AND A= 5.0 in². SOME POTENTIALLY HELPFUL INTERNAL LOADS HAVE BEEN GIVEN. USE VIRTUAL WORK. (20 POINTS)



b.) SUPPOSE THAT YOU WANTED TO ELIMINATE THIS DEFLECTION (I.E., MAKE $\Delta_{DY}=0$). ALSO SUPPOSE YOU WANTED TO DO SO BY HEATING UP MEMBER FG. HOW MUCH TEMPERATURE CHANGE WOULD YOU NEED IN FG TO MAKE Δ_{DY} ZERO AGAIN? (5 POINTS) a)

Member	F_Q (k)	F_P (k)	L (ft)	$F_P F_Q L$ (k^2ft)
AB	1	5	10	50
BC	1.4	7.1	$5\sqrt{2}$	70.6
~~CD~~	~~1.4~~	~~0~~		
~~DE~~	~~-1~~	~~0~~		
~~EF~~	~~-1~~	~~0~~		
FG	-2	-15	10	300
~~CE~~	~~0~~			
~~CF~~	~~0~~			
BF	-1	-5	10	50
AH	1.4	14	$5\sqrt{2}$	139
FH	1.4	14	$5\sqrt{2}$	139
~~BH~~	~~0~~			

$\Sigma = 748.9$ (k^2ft)

$$Q\,\delta_D = \Sigma \frac{F_P F_Q L}{AE} = \frac{(748.9)(12)}{(5)(29000)} = \boxed{0.062'' \downarrow}$$

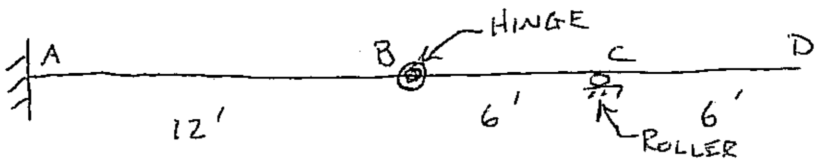
b.) $Q\,\delta_D = (F_{Q\,\overline{FG}})\,\alpha\,\Delta T\,L_{FG}$ (Assume α, if desired)

$1^k(.062'') = (-2)\,\alpha\,\Delta T\,(10')(12\text{ in/ft})$

$$\boxed{\Delta T = \frac{2.58\cdot 10^{-4}}{\alpha}}$$

PROBLEM 4.) INFLUENCE LINES

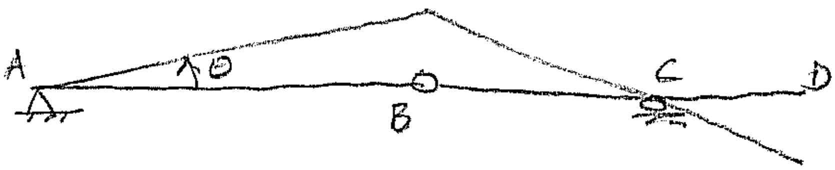
a.) FOR THE STRUCTURE BELOW, SKETCH THE INFLUENCE LINE DIAGRAM FOR THE MOMENT AT A. (15 POINTS)



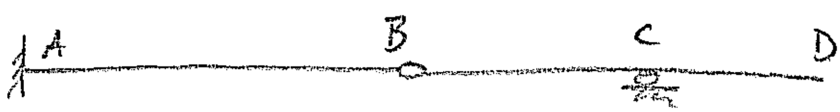
b.) TRICK QUESTION: FOR THE STRUCTURE ABOVE, SKETCH THE INFLUENCE LINE DIAGRAM FOR THE MOMENT AT B. (5 POINTS)

("SKETCH" INDICATES THAT NUMBERS ARE NOT REQUIRED, ONLY QUALATATIVE SHAPES.)

a.)



b.)



(No internal moment allowed at hinge. Influence line diagram is zero at all locations.)