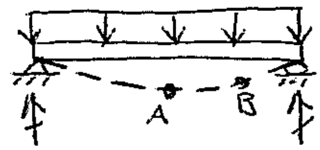
LESSON 11 DEFLECTION: MOMENT-AREA METHOD

READING: TEXT 9.3

→ CURVATURE: $\psi = \frac{d\theta}{dx} = \frac{M}{EI}$



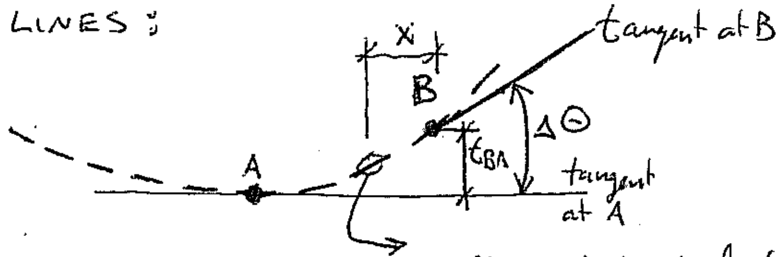
$$d\theta = \frac{M}{EI}\,dx$$

$$\Delta\theta_{AB} = \int_A^B d\theta = \int_A^B \frac{M}{EI}\,dx$$

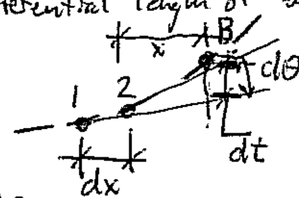
1) THE CHANGE IN SLOPE BETWEEN TWO POINTS ON A SMOOTH CONTINUOUS CURVE IS EQUAL TO THE AREA UNDER THE $\frac{M}{EI}$ CURVE.

(NOTE: SMOOTH IMPLIES NO HINGES)

→ TANGENT LINES:



Differential length of beam:



$$dt = (d\theta)\,x$$

$$dt = \frac{M}{EI}\,x\,dx$$

tangental deviation: $t_{BA} = \int_A^B dt = \int_A^B \frac{M}{EI}\,x\,dx$

t_{BA} → deviation of point B from tangent line at point A.

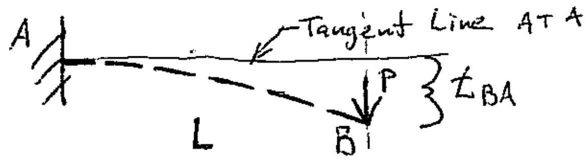
2) THE TANGENTAL DEVIATION AT A POINT ON A SMOOTH CONTINUOUS CURVE FROM A TANGENT LINE DRAWN AT A SECOND POINT ON THAT CURVE IS EQUAL TO THE MOMENT ABOUT THE FIRST POINT OF THE AREA UNDER THE $\frac{M}{EI}$ CURVE.

LIMITS: Shallow Curves
Elastic Behavior
Smooth Curves (No hinges)

- HOW TO APPLY MOMENT-AREA METHOD

  - DRAW MOMENT DIAGRAM
    (SHEAR DIAGRAM IS HELPFUL)
  - SKETCH THE DEFLECTED SHAPE
  - IDENTIFY POINTS OF KNOWN SLOPE:

EXAMPLE CANTILEVER BEAM

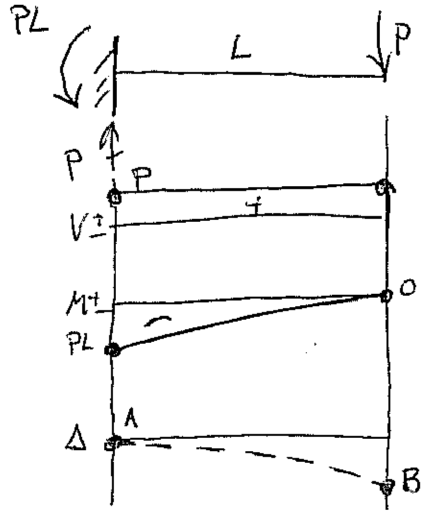


- The slope of a beam at a fixed-end support is zero; ∴ The tangent line is horizontal.

  → if we had an expression for t_{BA}, we could know the deflection at B!

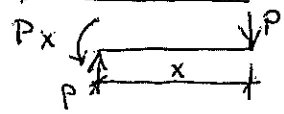
  → MOMENT-AREA METHOD:

$$t_{BA} = \int_A^B \frac{Mx}{EI}\,dx$$



EQN. Version

$M = P(x)$

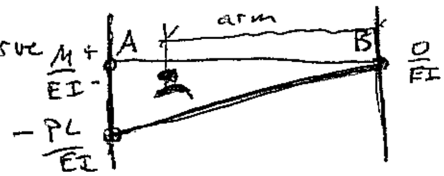


$$\int_A^B \frac{Mx}{EI}\,dx$$

$$\int_0^L \frac{(Px)x}{EI}\,dx$$

$$\frac{1}{EI}\left(\frac{1}{3}Px^3\right)\Big|_0^L = \frac{PL^3}{3EI}$$

⇒ $\frac{M}{EI}$ curve



→ IMAGINE $\frac{\text{MOMENT}}{EI}$ CURVE IS A DISTRIBUTED LOAD. WHAT IS ITS "MOMENT" AROUND B

SLOPE CHANGE
$\int_A^B \frac{M}{EI}\,dx$
AREA UNDER CURVE

→ $\text{AREA} = \frac{1}{2}\left(\frac{-PL}{EI}\right)(L)$

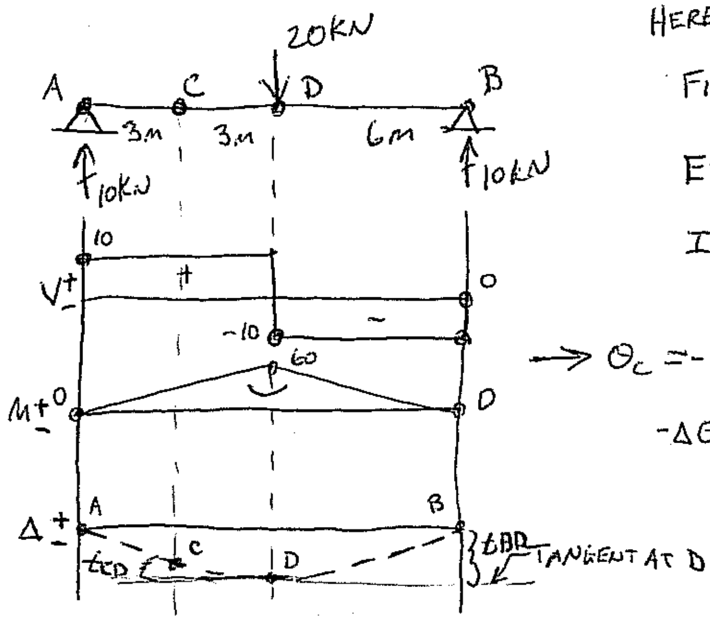
→ $\text{MOMENT ARM} = \frac{2L}{3}$

$$\therefore\ \Delta = t_{BA} = \text{"MOMENT"} = \frac{1}{2}\left(\frac{-PL^2}{EI}\right)\left(\frac{2L}{3}\right)$$

$$\Delta = -\frac{PL^3}{3EI}$$

CAN SOLVE GRAPHICALLY, SAVES TIME, INCREASE ACCURACY (MAYBE)

EXAMPLE & SYMMETRIC MEMBERS:



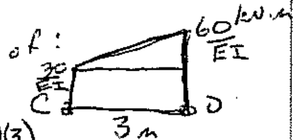
HERE, SLOPE @ D = 0

FIND Δ_D & Δ_C & Θ_C

$E = 200\ GPa = 200 \cdot 10^9 \frac{N}{m^2}$

$I = 6 \cdot 10^6\ mm^4 = 6 \cdot 10^{-6}\ m^4$

$\rightarrow \Theta_C = -\Delta\Theta_{CD} =$ Area of:



$-\Delta\Theta_{CD} = 30(3) + \frac{1}{2}(30)(3)$

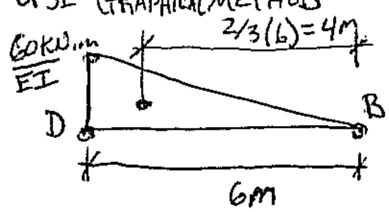
$= \frac{135}{EI}\ kNm^2$

$\boxed{\Theta_C = 0.113\ (rad.)}$

$\rightarrow \Delta_D = -t_{BD}$

$t_{BD} = \int_D^B \frac{M_x}{EI}\,dx$

USE GRAPHICAL METHOD



$t_{BD} = \underbrace{\frac{1}{2}(60)(6)}_{area}\ \underbrace{(4)}_{arm}\left(\frac{1}{EI}\right)$

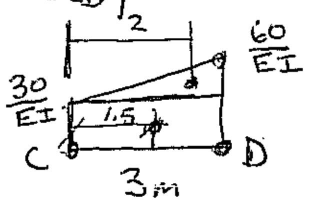
$t_{BD} = \frac{720}{EI}\ (kN\,m^3)$

$-\Delta_D = t_{BD} = 0.6\,m$

$\boxed{\Delta_D = -0.6\,m}$

$\rightarrow \Delta_C = -(t_{BD} - t_{CD})$

t_{CD}:



$t_{CD} = \frac{1}{2}\left(\frac{30}{EI}\right)(3)(2) + \left(\frac{30}{EI}\right)(3)(1.5)$

(60-30)

$t_{CD} = \frac{225\ kN\,m^3}{EI}$

$t_{CD} = 1.875\quad m = 0.19\,m$

$\Delta_C = -(0.6 - 0.19)$

$\boxed{\Delta_C = -.41\ m}$

MOMENT – AREA METHOD: THINGS TO KEEP IN MIND

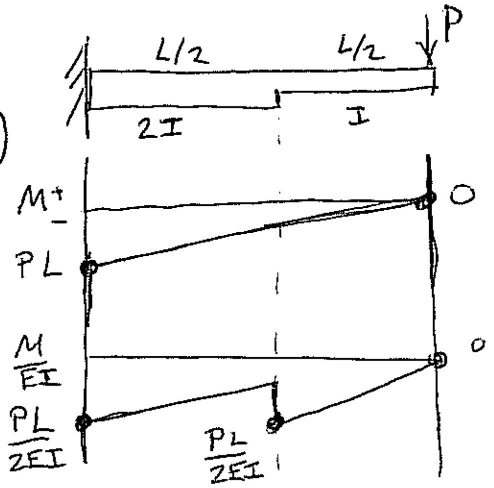
1.) NOTE SIGN OF $\frac{M}{EI}$ CURVE

M/EI	SLOPE	TANGENTAL DIFFERENCE
+ (A, B)	$\Delta\Theta_{AB}$ is positive $\Delta\Theta_{BA}$ is negative	t_{AB} pos., t_{BA} pos.
− (A, B)	$\Delta\Theta_{AB}$ IS NEGATIVE $\Delta\Theta_{BA}$ is positive	t_{AB} neg., t_{AB} neg.

2.) IF I VARIES IN BEAM, $\frac{M}{EI}$ CURVE IS AFFECTED

Ex.

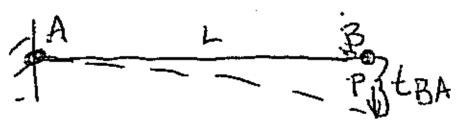
Also See Example 9.4



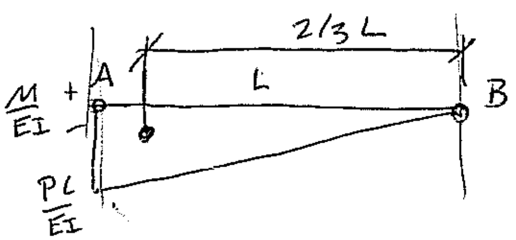
3.) DIRECTION OF "MOMENT ARM"

FOR t_{AB}, FIND "MOMENT" AROUND A

FOR t_{BA}, FIND "MOMENT" AROUND B



t_{BA} is tangent deviation at B from tangent of beam at A.



4.) Sometimes multiple applications are necessary to get deflection of interest.

5.) What is the difference between a definite and indefinite integral? Why am I asking you this?