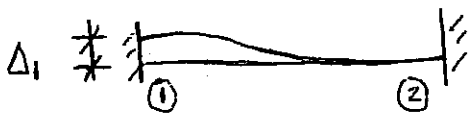
LESSON 29 : STIFFNESS MATRICIES FOR BEAMS

READING : TEXT, Ch. 18 {NEGLECT FOR EXAM3}

- POTENTIAL DOFS AND FORCES
- MEMBER STIFFNESS MATRIX, 4x4
- AXIAL EFFECTS

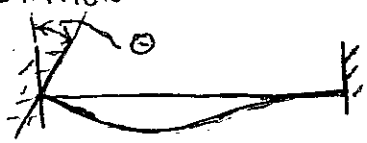
A.) POTENTIAL DOFS & FORCES

1.) DISPLACEMENT



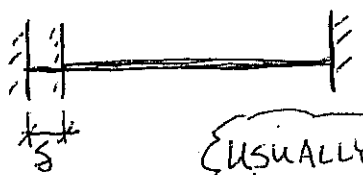
CONSIDER AT EACH END

2.) ROTATION



CONSIDER AT EACH END

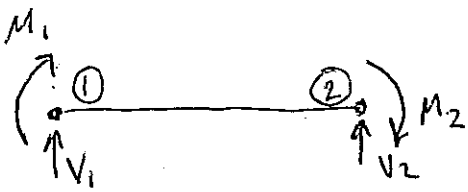
3.) AXIAL DEFLECTION



{USUALLY NEGLECT IN BEAMS}

ALSO.) TORSION, SHEAR DEFORMATION } TYPICALLY NEGLECT

TAKING INTO ACCOUNT DISPLACEMENT & ROTATION WE CONSIDER THE FOLLOWING FORCES :



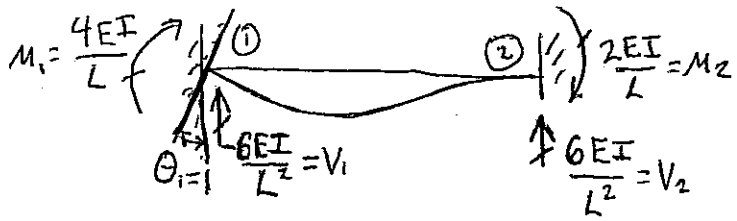
4 DOFS $\Leftrightarrow$ 4 END FORCES

STIFFNESS MATRIX IS 4x4

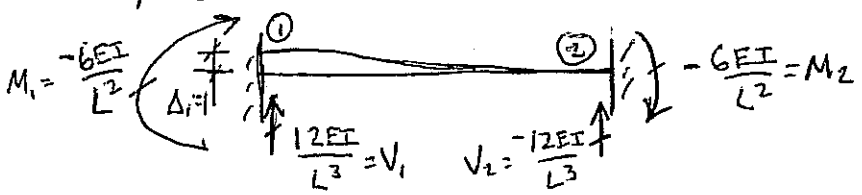
B.) DERIVATION OF MEMBER STIFFNESS MATRIX

APPLY EACH DISP. AND SOLVE FOR END FORCES:

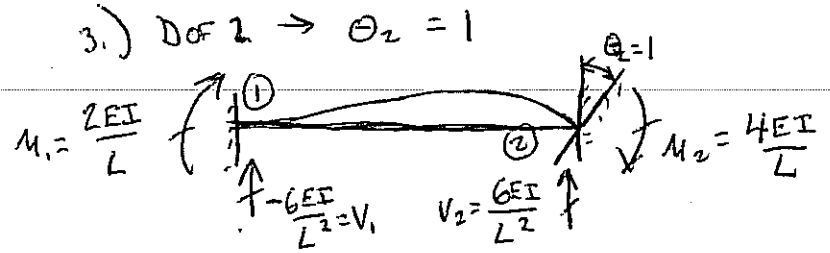
1.) DOF 1 → $\theta_1 = 1$



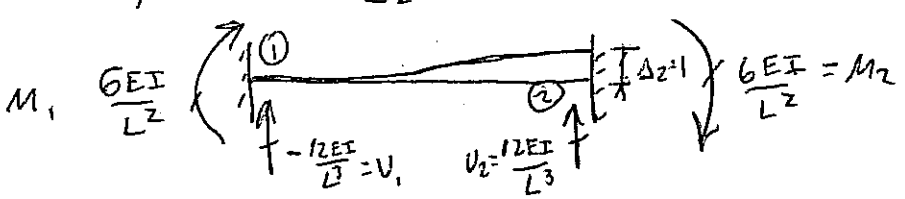
2.) DOF 3 → $\Delta_1 = 1$



3.) DOF 2 → $\theta_2 = 1$



4.) DOF 4 → $\Delta_2 = 1$



$$\begin{bmatrix} M_1 \\ M_2 \\ V_1 \\ V_2 \end{bmatrix} = \begin{matrix} \\ \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix} \begin{matrix} \begin{matrix} \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \end{matrix} \\ \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{6EI}{L^2} \\ \frac{2EI}{L} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{12EI}{L^3} \\ \frac{6EI}{L^2} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{12EI}{L^3} \end{bmatrix} \end{matrix} \begin{matrix} \\ \textcircled{1} \\ \textcircled{2} \\ \textcircled{3} \\ \textcircled{4} \end{matrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \Delta_1 \\ \Delta_2 \end{bmatrix}$$

$$\bar{q}_{beam} = \underset{\uparrow 4\times 4}{\bar{K}_{beam}} \; \bar{\Delta}_{beam}$$

NOTE: STIFFNESS MATRIX IS SYMMETRIC.

C.) AXIAL EFFECTS:

AUGMENT M & V SYSTEM W/ AXIAL FORCES & DISP.

$$\begin{bmatrix} M_1 \\ M_2 \\ V_1 \\ V_2 \\ \hline F_1 \\ F_2 \end{bmatrix} = \left[\begin{array}{cccc|cc} \frac{4EI}{L} & \frac{2EI}{L} & \frac{-6EI}{L^2} & \frac{6EI}{L^2} & 0 & 0 \\ \frac{2EI}{L} & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{6EI}{L^2} & 0 & 0 \\ \frac{-6EI}{L^2} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} & \frac{-12EI}{L^3} & 0 & 0 \\ \frac{6EI}{L^2} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{12EI}{L^3} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \frac{AE}{L} & \frac{-AE}{L} \\ 0 & 0 & 0 & 0 & \frac{-AE}{L} & \frac{AE}{L} \end{array}\right] \begin{bmatrix} \theta_1 \\ \theta_2 \\ \Delta_1 \\ \Delta_2 \\ \hline \delta_1 \\ \delta_2 \end{bmatrix}$$

↖ 6x6 stiffness matrix with axial effects

$M/\theta/V/\Delta \Longleftrightarrow F\delta$

(UNCOUPLED)

- ALLOWS US TO CONSIDER SEPARATELY:
- TORSION IS SIMILARLY UNCOUPLED.

- MATRIX METHOD IN 3D's

MEMBER STIFFNESS MATRIX IS 12x12