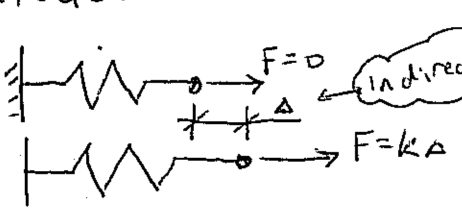
LESSON 25 DIRECT STIFFNESS FORMULATION FOR TRUSS ELEMENTS

READING: TEXT Ch. 17

LESSON OVERVIEW:

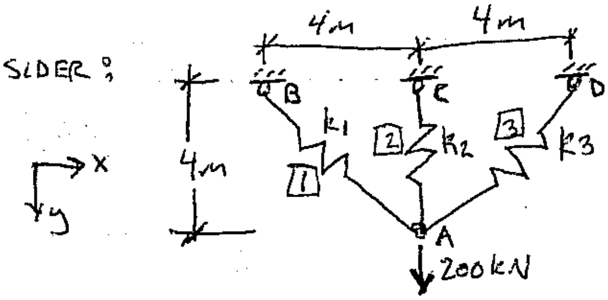
- FUN WITH SPRINGS
- CONNECTION TO TRUSS ELEMENTS
- CHANGEING COORDINATE NAMES

A.) SPRINGS:



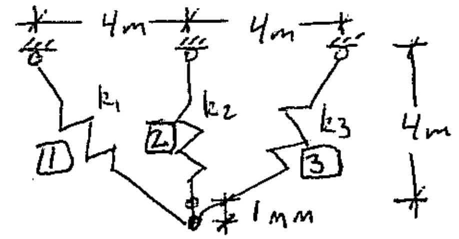
where k = spring constant

- CONSIDER:



$k_1 = 35.4$ kN/mm
$k_2 = 25$ kN/mm
$k_3 = 17.7$ kN/mm

- What happens when we create a unit deformation At joint A, in the pos. y-direction?



FORCES DUE TO $\Delta_y = 1$

$\Delta L_1 = 0.707$ mm
$\Delta L_2 = 1$ mm
$\Delta L_3 = 0.707$ mm

Therefore, $F_{1y} = k_1 \Delta L_1 = 25.0$ kN
$F_{2y} = k_2 \Delta L_2 = 25$ kN
$F_{3y} = k_3 \Delta L_3 = 12.5$ kN
(FOR A UNIT DISP. +y-dir.)

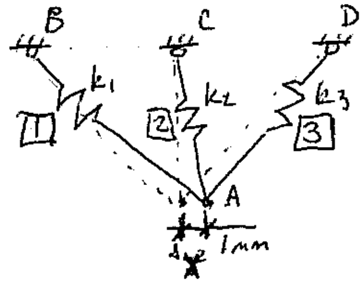
we can say, in general, for $\Delta y \neq$ unit deformation:

$F_{1y} = 25.0\,\Delta_y$ ← UNITS? kN/mm

$F_{2y} = 25.0\,\Delta_y$

$F_{3y} = 12.5\,\Delta_y$

- What about a unit deflection in the +x-direction?



FORCES DUE TO $\Delta_x = 1$

$\Delta L_1 = 0.707\,mm$
$\Delta L_2 = 0 \leftarrow$ (SMALL DEFL. THEORY!)
$\Delta L_3 = -0.707\,mm$ (COMPRESSION)

$F_{1x} = k_1 \Delta L_1 = 25.0\,kN$
$F_{2x} = k_2 \Delta L_2 = 0$
$F_{3x} = k_3 \Delta L_3 = -12.5\,kN$
(FOR A UNIT DISP. +x-dir.)

We can say, in general:

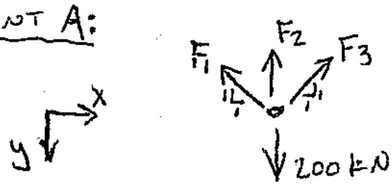
$$F_{1x} = 25.0\,\Delta_x$$
$$F_{2x} = 0\,\Delta_x$$
$$F_{3x} = -12.5\,\Delta_x$$

- By superposition (Assuming Linear-Elastic Behavior)

EQN. ①
$$\begin{cases} F_1 = F_{1x} + F_{1y} = 25.0\Delta_x + 25.0\Delta_y \\ F_2 = F_{2x} + F_{2y} = 0\,\Delta_x + 25.0\,\Delta_y \\ F_3 = F_{3x} + F_{3y} = -12.5\Delta_x + 12.5\Delta_y \end{cases}$$

- But what are the real values of Δ_x & Δ_y???
- CONSIDER EQUILIBRIUM AT THE FREE JOINT:

JOINT A:



EQN. ②
$$\begin{cases} \Sigma F_y = 0]\; -\frac{1}{\sqrt{2}} F_1 - F_2 + \frac{-1}{\sqrt{2}} F_3 = -200 \\ \Sigma F_x = 0]\; -\frac{1}{\sqrt{2}} F_1 + \frac{1}{\sqrt{2}} F_3 = 0 \end{cases}$$

PLUG ① INTO ②:

$$-\frac{1}{\sqrt{2}}(25.0\Delta_x + 25.0\Delta_y) - 25\Delta_y - \frac{1}{\sqrt{2}}(-12.5\Delta_x + 12.5\Delta_y) = -200$$
$$-\frac{1}{\sqrt{2}}(25.0\Delta_x + 25.0\Delta_y) + \frac{1}{\sqrt{2}}(-12.5\Delta_x + 12.5\Delta_y) = 0$$

2 EQNS, 2 UNKNOWNS!

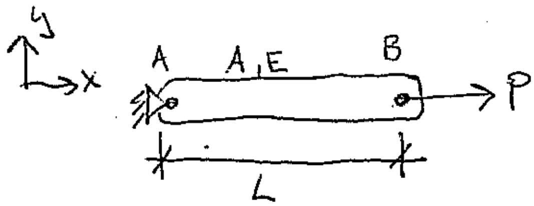
After Some SIMPLIFICATIONS:

$$-8.84\Delta_x - 51.52\Delta_y = -200$$
$$-26.52\Delta_x - 8.84\Delta_y = 0$$

$\Delta_x = -1.37\,mm \leftarrow$
$\Delta_y = 4.12\,mm \downarrow$

FORCES IN SPRINGS: $F_1 = 25.0(-1.37) + 25.0(4.12) = 68.7\,kN$ (T)
$F_2 = 0\,(-1.37) + 25.0(4.12) = 103\,kN$ (T)
$F_3 = -12.5(-1.37) + 12.5(4.12) = 68.6\,kN$ (T)

B.) CONNECTION OF SPRINGS TO TRUSS ELEMENTS



WHAT IS δ_{BX}?

VIRTUAL WORK: $F_P = P$, $F_Q = 1$

$$\cancel{\mathscr{Q}}\,\Delta_{BX} = \frac{F_P \cancel{F_Q} L}{AE}$$

$$\Delta_{BX} = P\frac{L}{AE}$$

Looks LIKE SPRING EQN.

Rearrange: $P = \frac{AE}{L}\Delta_{BX}$

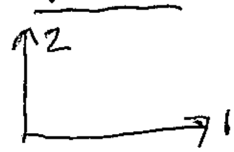
Spring EQN. $F = k\,\Delta$ where $\boxed{k = \frac{AE}{L}}$

- DIRECT STIFFNESS METHOD TREATS BEAM MEMBERS AS SPRINGS w/ STIFFNESS AE/L.

C.) COORDINATE NAMES

<u>UP TILL NOW</u>:

<u>FUTURE</u>: (YOU MAY SEE)



- Necessary Because Stiffness Method is Matrix-Based
- We will treat directions as indices in a matrix.
- See it in action, next time...

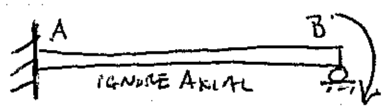
D.) KINEMATIC INDETERMINACY (Text 12.6)

STATIC INDETERMINACY: OUR FOCUS TILL NOW

- BASED ON → # UNKNOWN FORCES − # EQNS. OF STATIC EQUILIBRIUM

KINEMATIC INDETERMINACY:

- BASED ON → # OF DEGREES OF FREEDOM ALLOTTED TO JOINTS (DOF)
- DICTATES THE NUMBER OF DEFORMATIONS YOU NEED TO CONSIDER IN DIRECT STIFFNESS METHOD

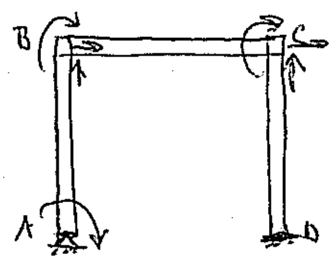


JOINTS

A: 2 POTENTIAL DOF, ALL RESTRAINED
B: 2 POTENTIAL DOF, 1 RESTRAINED

K.I. = 1 degree

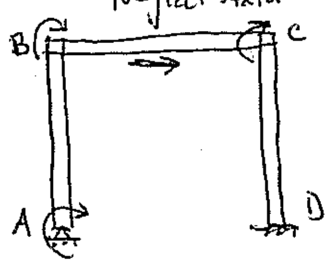
CONSIDER AXIAL



JOINTS: A → 1 degree
B → 3 degrees
C → 3 degrees
D → 0 degrees

$\left.\right\}$ K.I. 7 degrees

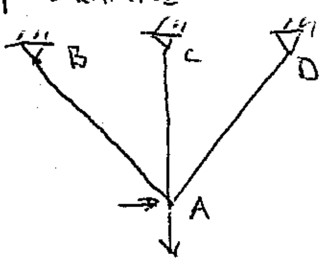
Neglect Axial



JOINTS: A → 1 degree
B → 1 degree
C → 1 degree
D → 0 degrees
B & C → 1 degree

$\left.\right\}$ K.I. 4 degrees

SPRING (TRUSS) EXAMPLE:



JOINTS

A: 2 degrees
B: 0 deg
C: 0 deg
D: 0 deg

$\left.\right\}$ K.I. 2 deg.