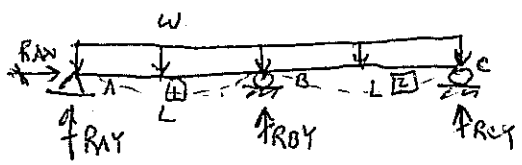
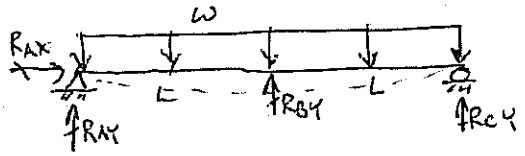
Lesson 21 Flexibility Examples

EX 1



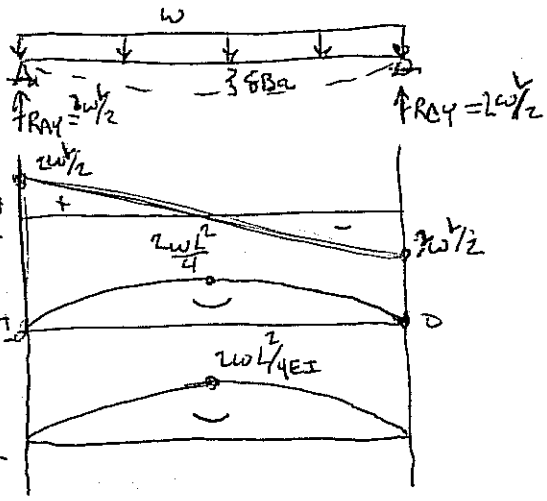
I EI
2 EI

Draw released structure:



APPLIED LOADS:

a)



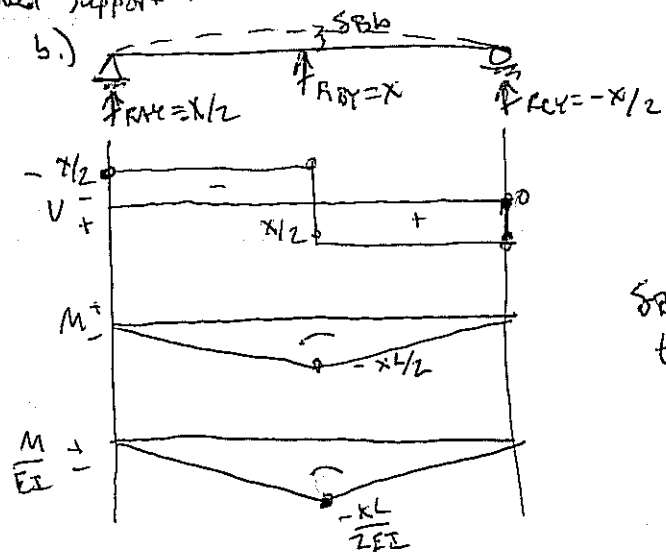
δ_{Ba} from Moment-Area method

$t_{AB} = \frac{2}{3}\left(\frac{2wL^2}{4EI}\right)L\left(\frac{5L}{8}\right) = \frac{5wL^4}{24EI}$

$\delta_{Ba} = -t_{AB} = -\frac{5wL^4}{24EI}$

Removed Support Reactions

b.)



δ_{Bb}:

$t_{AB} = \frac{1}{2}\left(-\frac{XL}{2EI}\right)(L)\left(\frac{2L}{3}\right)$

$\delta_{Bb} = -t_{AB} = \frac{XL^3}{6EI}$

EX 1 CONT.

Now ADD CONTRIBUTIONS FROM a & b and set equal to true deflection:

$$\frac{-5wL^4}{24EI} + \frac{XL^3}{6EI} = 0$$

$$R_{BY} = X = \frac{5}{4}wL = 1.25wL$$

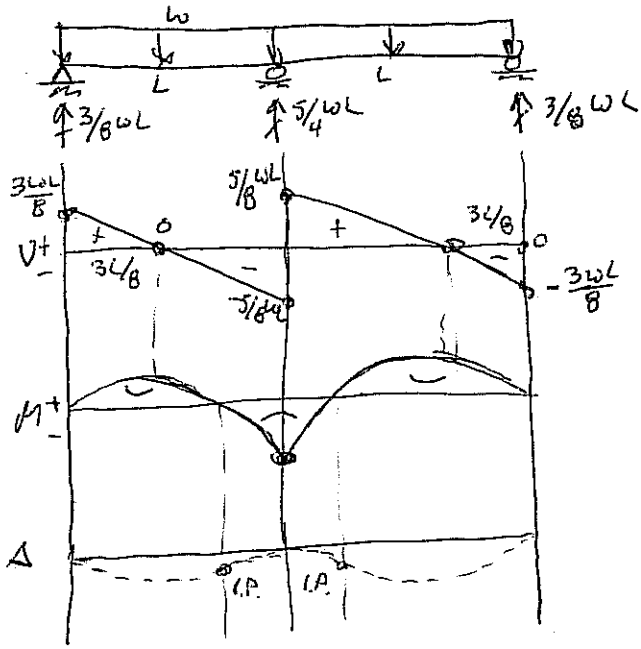
Now, solve for other reactions

$\Sigma F_y = 0]$ $R_{AY} + R_{CY} + 1.25wL = 2wL$

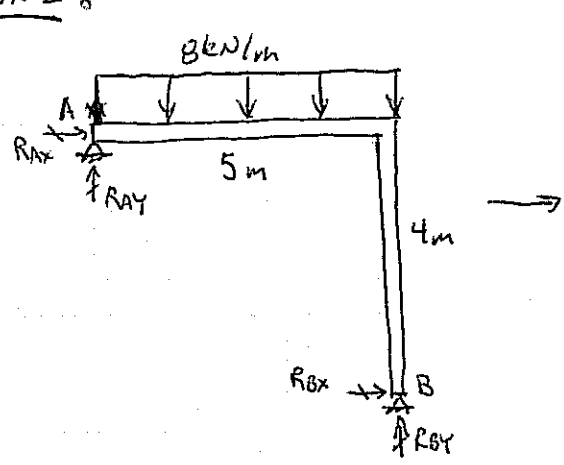
$\circlearrowleft + \Sigma M_A = 0] + (1.25wL)(L) + 2LR_{CY} - 2wL^2 = 0$

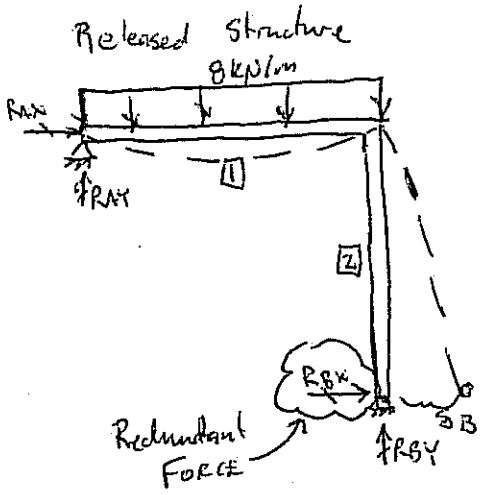
$$R_{CY} = wL - 5/8\,wL = 3/8\,wL$$

$$R_{AY} = 3/8\,wL$$



Ex 2:

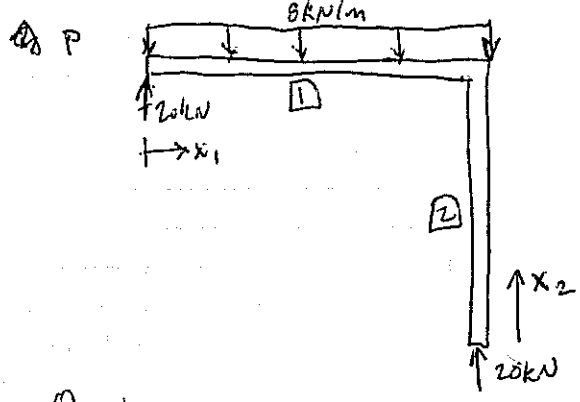




Indeterminate: 1st Degree

a.) δ_B due to applied loads; use Virtual work:

Book will call this "Primary Structure"

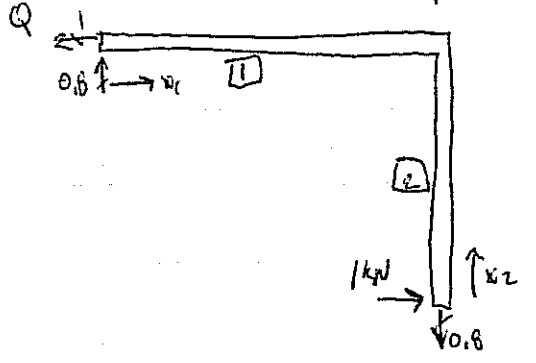


1 $M_1 = 20x_1 - 4x_1^2$

2 $M_2 = 0$

$$\delta_{Ba} = \int_0^L \frac{M_Q M_P}{EI} dx = \int_0^5 \frac{(20x_1 - 4x_1^2)(0.8x_1)}{EI} dx_1$$

$$= \frac{166.7}{EI}$$

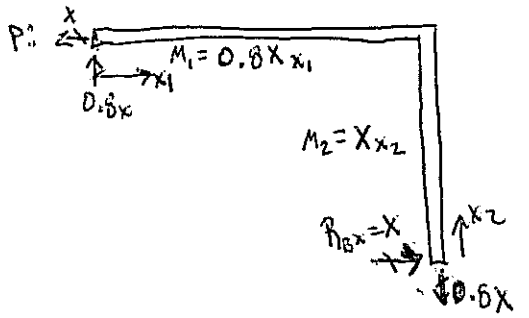


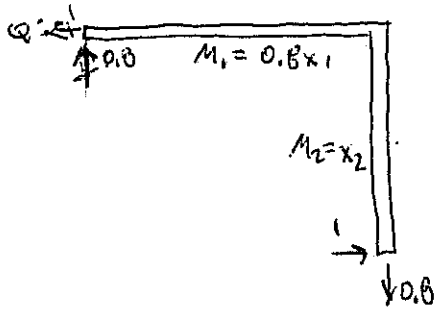
1 $M_1 = 0.8x_1$

$M_2 = 1x_2$

b.) δ_B due to Redundant force:

Book will call this a "Secondary Structure"





$$\delta_{Bb} = \int_0^L \frac{M_P M_Q}{EI} dx$$

$$= \int_0^5 \frac{(0.8x_1)^2 X}{EI} dx_1 + \int_0^4 \frac{(x_2)^2 X}{EI} dx_2$$

$$\delta_{Bb} = \frac{48.0\,X}{EI}$$

EX 2 CONT.

→ Add a & b deflections and set to true value:

$$\frac{166.7}{EI} + \frac{48X}{EI} = 0$$

$$R_{BX} = X = -3.47\ kN$$

$\rightarrow \quad \Sigma F_x = 0]\ R_{Ax} - 3.47\ kN = 0 \rightarrow R_{Ax} = 3.47\ kN$

$\Sigma F_y = 0]\quad R_{AY} - 8(5) + R_{BY} = 0$

$\circlearrowleft + \quad \Sigma M_A = 0]\ -40(2.5) + R_{BY}(5) - 3.47(4) = 0$

$R_{BY} = 22.8\ kN$

$R_{AY} = 17.2\ kN$