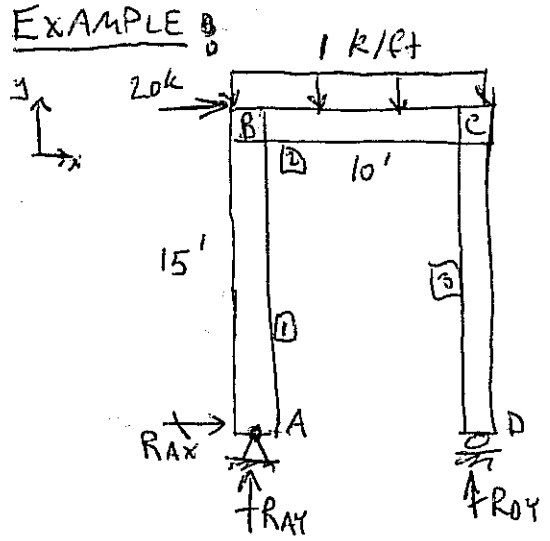
# LESSON 15 - VIRTUAL WORK IN FRAMES

FRAME MEMBERS MAY HAVE MOMENTS & AXIAL LOADS:
- NEED TO ACCOUNT FOR ALL EFFECTS: (except when we don't)
- FRAMES ARE MADE FROM MULTIPLE MEMBERS, (LIKE TRUSSES)
  - NEED TO SUM INTERNAL STRAIN ENERGY FROM ALL MEMBERS (LIKE TRUSSES)
  - Member internal forces change over length, need to integrate (LIKE BEAMS)

## EXAMPLE




FIND $\delta_{CX}$

$E = 29000$ ksi
$I = 600$ in$^4$
$A = 13$ in$^2$
} ALL MEMBERS

$R_{AX} = -20k$
$R_{AY} = -25^k$
$R_{DY} = 35^k$

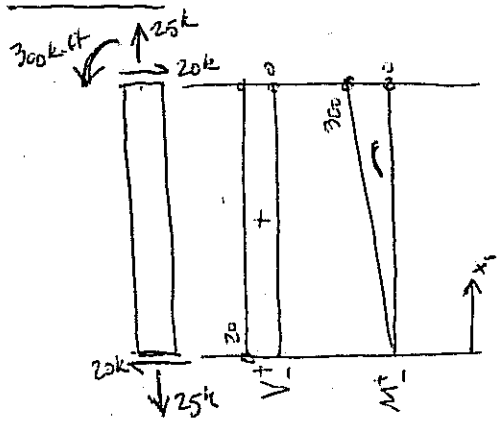
$\circlearrowleft \Sigma M_A = 0] -20(15) - 10(5) + R_{DY}(10) = 0$

$\Sigma F_y = 0] -(10)(1) + R_{AY} + R_{DY} = 0$

$\Sigma F_x = 0] 20 + R_{AX} = 0$

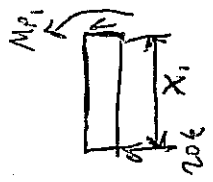
## P-SYSTEM

Member 1




$M_{P_1} = 20 x_1 (k)$

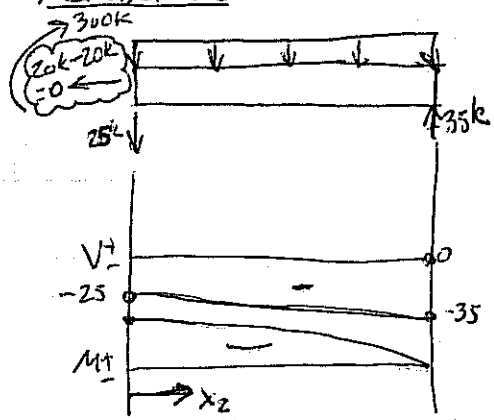
$F_{P_1} = 25 (k)$ (Tension)

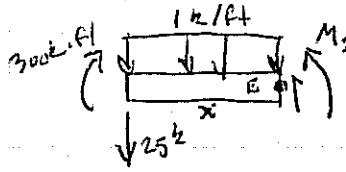


... Ex. Cont.

P-SYSTEM

MEMBER 2




$\curvearrowleft \Sigma M_E = 0$

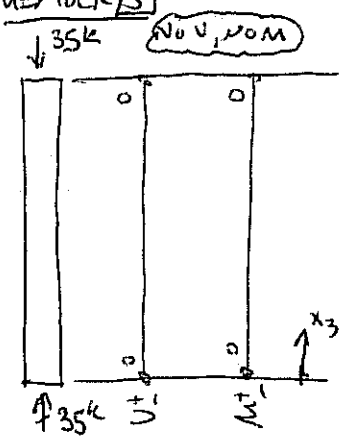
$M_2 - 300 + \frac{x^2}{2} + 25x = 0$

$M_2 = -\frac{x^2}{2}(k/ft) - 25x(k) + 300\,k\cdot ft$

$M_{P_2} = M_2 = -\frac{x^2}{2}(k/ft) - 25x(k) + 300\,k\cdot ft$

$F_{P_3} = 0\,k$

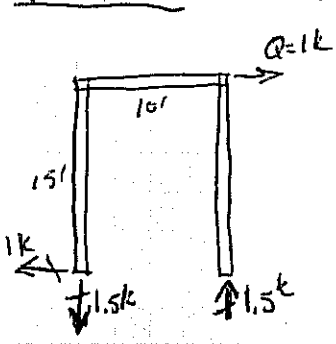
MEMBER 3



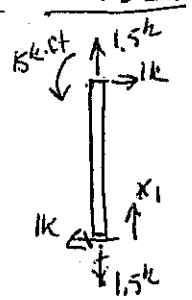

$M_{P_3} = 0$

$F_{P_3} = -35\,k$ (comp.)

Q-SYSTEM (δCx)



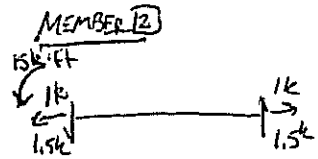

Member 1




$M_{Q_1} = 1x_1(k)$

$F_{Q_1} = 1.5\,k$ (Ten.)

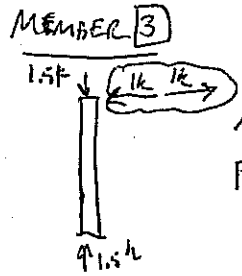
MEMBER 2




$M_{Q_2} = -1.5x(k) + 15\,k\cdot ft$

$F_{Q_2} = 1^k$ (T)

MEMBER 3




$M_{Q_3} = 0$

$F_{Q_3} = -1.5k$ (Comp)

→ PUT IT TOGETHER (NEXT PG.)

Ex. Cont.

$$Q\delta = \sum \frac{F_Q F_P L}{AE} + \sum \int_0^L M_Q \frac{M_P}{EI}\, dx$$

(1k) $\delta_{Cx} = \dfrac{(1.5\,k)(25\,k)(15')(12\,in/ft)}{(13\,in^2)(29000\,ksi)}$ + $\dfrac{(1.0k)(0k)(10')(12\,in/ft)}{(13\,in^2)(29000\,ksi)}$ + $\dfrac{(-1.5k)(-35k)(15')(12\,in/ft)}{(13\,in^2)(29000\,ksi)}$ + ...

Axial 1 — Axial 2 (0) — Axial 3

... + $\int_0^{15} \dfrac{x_1(k)\,20\,x_1(k)}{(600\,in^4)(29000\,ksi)}\,dx_1$ + $\int_0^{10} \dfrac{[-1.5x_2(k)+15\,k\cdot ft]\left[-\frac{x_2^2}{2}(k/ft)-25\,x_2(k)+300\,k\cdot ft\right]}{(600\,in^4)(29000\,ksi)}\,dx_2$ + ...

Moment 1 — Moment 2

... + $\int_0^{15} \dfrac{(0)(0)}{600(29000)}\,dx_3$ = 0

Moment 3

(1k) $\delta_{Cx} = 0.043'' + \dfrac{20\,x_1^3 (k^4)(12)^3}{(3)1.74\cdot10^7\,(k\cdot in^2)}\Big|_{0'}^{15'} + \dfrac{0.1875\,x_2^4 + 10x_2^3 - 387.5x_2 + 4500x_2}{1.74\cdot10^7\,(k\cdot in^2)}\Big|_{0'}^{10'} (12\,in/ft)^3$

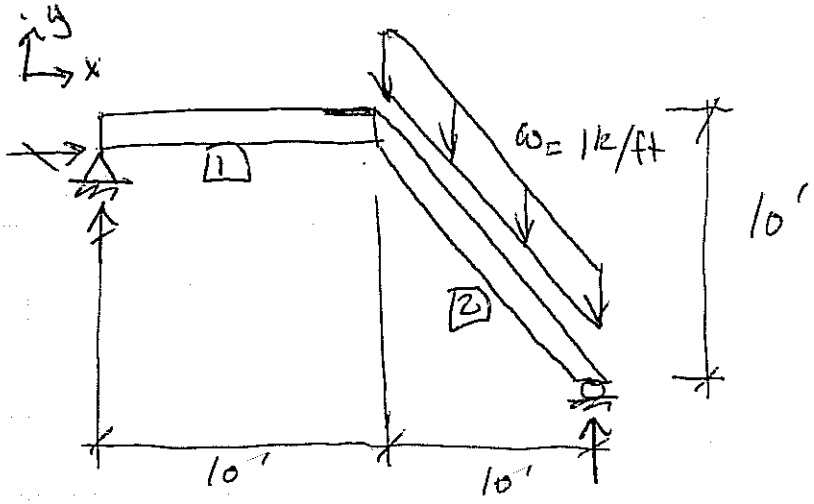
Axial terms → 0.043"; M1 → 2.23"; M2 → 1.80"

$\delta_{Cx} = \underbrace{0.043''}_{\text{Axial terms}} + \underbrace{2.23'' + 1.80''}_{\text{Moment Terms}}$

$$\boxed{\delta_{Cx} = 4.07''}$$

NOTE: Axial effects pretty insignificant.

SOMETHING TO THINK ABOUT:




What is AXIAL STRAIN ENERGY IN 2
FOR P-SYSTEM

$$\frac{F_Q F_P L}{A E} \rightarrow \int \frac{F_Q F_P L}{AE} dx$$

INTEGRALS HERE TOO!