# LESSON 13 – VIRTUAL WORK METHOD – TRUSSES

BY APPLYING DUMMY LOADS,
DEFLECTIONS ANYWHERE IN A STRUCTURE
MAY BE DETERMINED

→ DUMMY LOADS: FALSE LOAD APPLIED TO STRUCTURE
– LOCATION OF DESIRED DISP.
– DIRECTION OF DESIRED DISP.

– CUSTOMARY TO USE UNIT LOAD (e.g., 1 kip)
– KEY: DISPLACEMENTS IT UNDERGOES,
CAUSED BY REAL LOADS

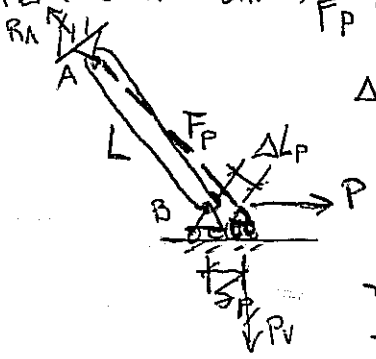
(Q-SYSTEM IN TEXT)

→ REAL LOADS: – FORCES & MOMENTS APPLIED TO STRUCTURE
– ALSO, EFFECTS OF:
– TEMP. CHANGE
– SUPPORT SETTLEMENT
– MEMBER OVER/UNDER SIZE

(P-SYSTEM IN TEXT)

→ Application to truss Members:

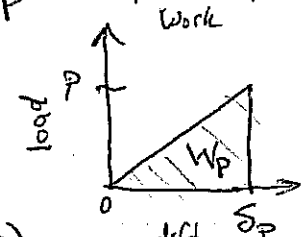
P-SYSTEM (REAL LOADS)

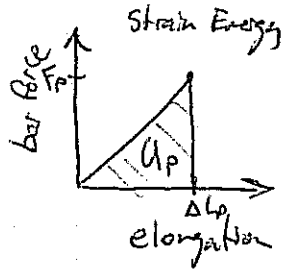



$F_P$: internal force due to P-SYSTEM

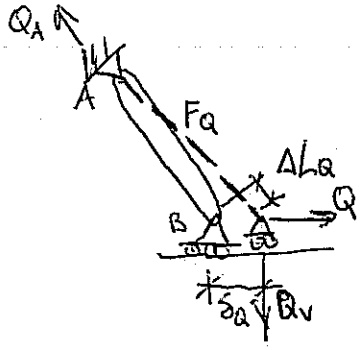
$\Delta L_P$: Change in member length due to P-SYSTEM

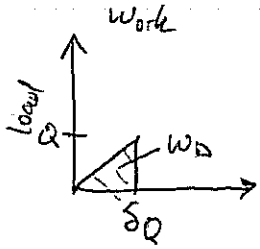
$\delta_P$: Displacement (horiz.) due to P-SYSTEM

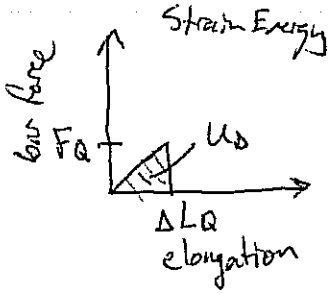


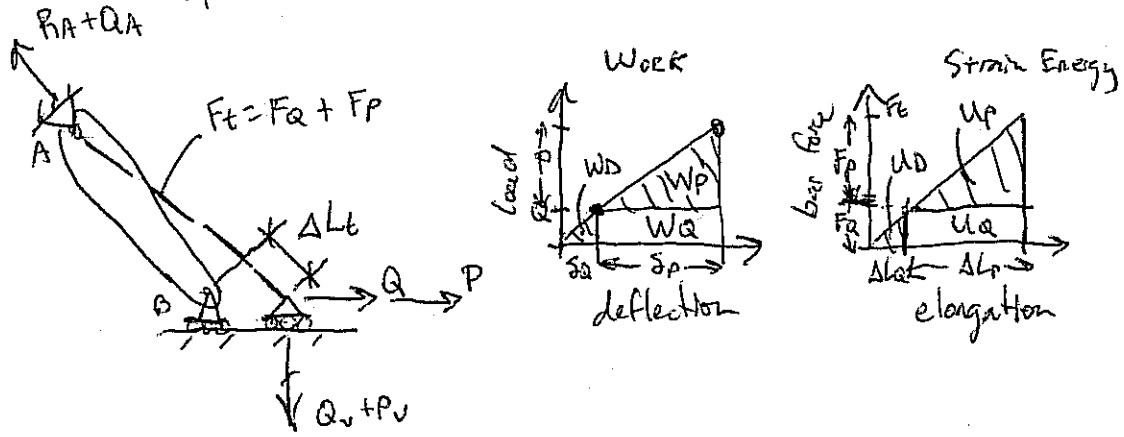

Q-SYSTEM (DUMMY LOAD)



P/Q: FIRST APPLY Q, THEN APPLY P




**P-SYSTEM**

$$\Delta L_P = \frac{F_P L}{AE} \quad (3)$$

$W_P = \frac{1}{2} P \delta_P$

$U_P = \frac{1}{2} F_P \Delta L_P$

$W_P = U_P$

**Q-SYSTEM**

$$\Delta L_Q = \frac{F_Q L}{AE}$$

$W_D = \frac{1}{2} Q \delta_Q$

$U_D = \frac{1}{2} F_Q \Delta L_Q$

$W_D = U_D$

**P/Q SYSTEM**

$$\Delta L_t = \frac{(F_Q + F_P)L}{AE}$$

$$W_Q = Q \delta_P \quad (2)$$

$$U_Q = F_Q \Delta L_P$$

$$W_Q = U_Q \quad (1)$$

→ SINGLE BAR; TRUSS: $Q \delta_P = F_Q \Delta L_P$ ← SUB. (2) into (1)

→ GENERAL TRUSS; (SUM ALL MEMBERS): $\sum Q \delta_P = \sum F_Q \Delta L_P$ (NOTE NOTATION IN CLASS)

SUB. (3) → $\sum Q \delta_P = \sum F_Q \frac{F_P L}{AE}$

$$\sum Q \delta_P = \sum \frac{F_Q F_P L}{AE}$$

Q = DUMMY LOAD(S)

$\delta_P$ = DISPLACEMENT DUE TO REAL LOADS (IN DIRECTION OF Q)

$F_Q$ = BAR FORCE(S) DUE TO DUMMY LOAD(S)

$F_P$ = BAR FORCES, DUE TO REAL LOAD(S)

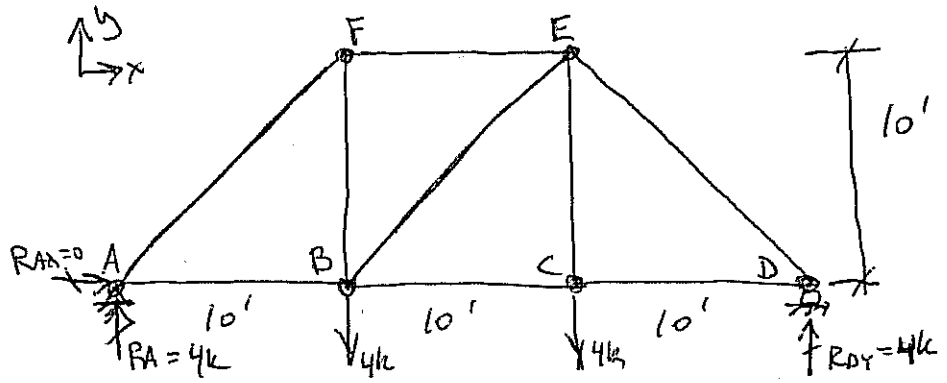
L = MEMBER LENGTH(S)

A = MEMBER AREA(S)

E = MEMBER YOUNG'S MODULUS (II)

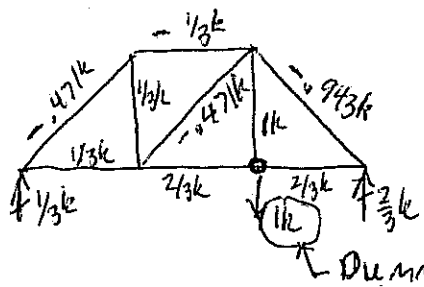
→ MULTIPLE DEFLECTIONS/DIRECTIONS? → MULTIPLE APPLICATIONS. IN LAB

EXAMPLE FIND Vert. Disp. @ C due to loads shown.




$A = 0.5\ in^2$
$E = 29 \cdot 10^3\ ksi$

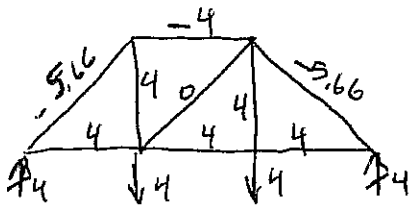
Q SYSTEM:




DUMMY LOAD, Q
DOWN, AT C

NOTE SIGNS!
- = COMP,
+ = TEN,

P SYSTEM




| Member | $F_Q$ (k) | $F_P$ (k) | L (ft) | AE (k) | $F_Q F_P L$ ($k^2$-ft) |
|---|---|---|---|---|---|
| AB | .333 | 4 | 10 | $14.5 \cdot 10^3$ | 13.33 |
| BC | .667 | 4 | 10 | $14.5 \cdot 10^3$ | 26.67 |
| CD | .667 | 4 | 10 | $14.5 \cdot 10^3$ | 26.67 |
| DE | -.943 | -5.66 | 14.14 | $14.5 \cdot 10^3$ | 75.42 |
| FE | -.333 | -4 | 10 | $14.5 \cdot 10^3$ | 13.33 |
| EB | -.471 | 0 | 14.14 | $14.5 \cdot 10^3$ | 0 |
| BF | .333 | 4 | 10 | $14.5 \cdot 10^3$ | 13.33 |
| AF | -.471 | -5.66 | 14.14 | $14.5 \cdot 10^3$ | 37.71 |
| CE | 1 | 4 | 10 | $14.5 \cdot 10^3$ | 40 |
| | | | | factors out | $\Sigma = 246.47\ k^2 \cdot ft$ |

$$(1^k)\Delta_{Cy} = \frac{(246.47\ k^2\text{-ft})(12\ in/ft)}{14.5 \cdot 10^3\ k}$$

$$\boxed{\Delta_{Cy} = 0.204\ in}$$

## CAN ALSO INCLUDE OTHER EFFECTS:

### TEMPERATURE CHANGE:

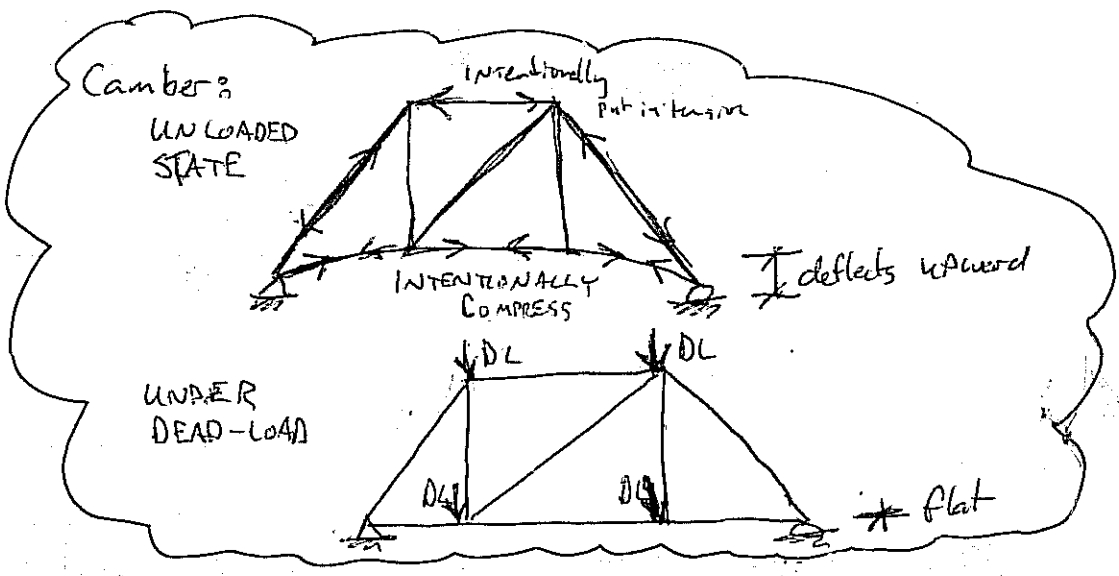
$$\sum Q\,\delta = \sum F_Q\,\alpha(\Delta T)\,L$$

$\alpha$ = coefficient of thermal expansion
$\Delta T$ = change in temperature

### FABRICATION ERROR / CAMBER:

$$\sum Q\,\delta = \sum F_Q\,\Delta L$$

$\Delta L$ = difference in length of a member from its intended size.
(Longer = +, Shorter = −)

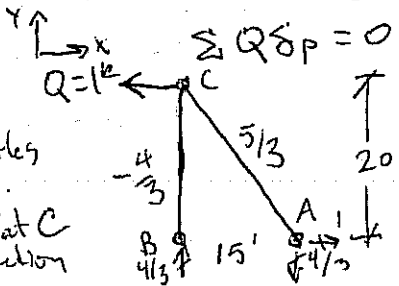



### SUPPORT SETTLEMENT:

IN DETERMINATE STRUCTURES, SUPPORT SETTLEMENTS CAUSE NO INTERNAL LOADING:

EX:

JOINT A settles 0.6" DOWN. what is defl. at C in x direction




$\sum Q\delta_P = 0$

$(1)(\delta_{Cx}) + (\frac{4}{3})(.6)$

$\delta_{Cx} = 0.8$ in

→ YOU CAN COMBINE THESE EFFECTS WITH LOADS OR WITH EACH OTHER!