WORK-ENERGY METHODS FOR DEFLECTIONS

READING: TEXT CHAPTER 10

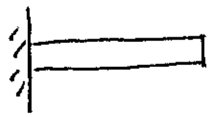
Definition of work due to force

WORK = FORCE × DISPLACEMENT ← (IN DIRECTION OF FORCE (VECTOR QUANTITIES)

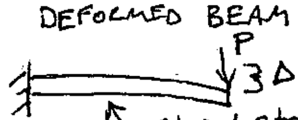
Conservation of energy:

Applied External Work = STORED INTERNAL ENERGY

UNDEFORMED BEAM



DEFORMED BEAM



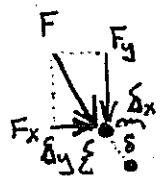
PΔ ≅ external work

← stored strain energy

CE 3202: LOAD APPLIED SLOWLY → NO KINETIC ENERGY (NOT DYNAMIC)
NO HEAT ENERGY (NO DAMPING)

A. WORK DONE BY FORCES: (W)

FORCES



$W = F\delta$

$W = F_x\delta_x + F_y\delta_y$

- Displacement is in direction of Force.

MOMENTS

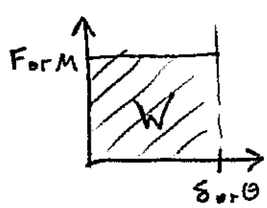
$W = M\Theta$

- The above assume that forces Remain constant as they are applied:

IN GENERAL: $W = \int_0^\delta F\,d\delta$ For forces

$W = \int_0^\Theta M\,d\theta$ for moments

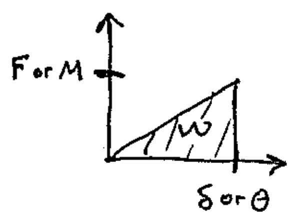
Constant Case:



$W = F\delta$

$W = M\theta$

Linearly Varying case (Gradually Applied Load)



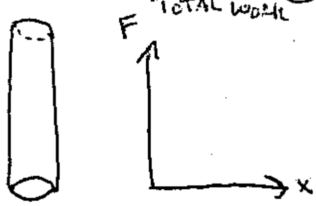
$W = \frac{1}{2}F\delta$

$W = \frac{1}{2}M\Theta$

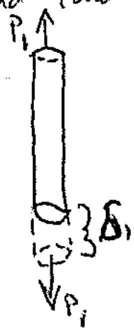
(Which one to use???)

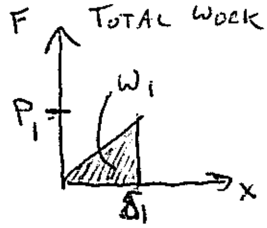
IMPORTANT DISTINCTION:

① Unloaded bar



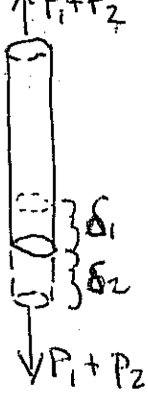
② Add load gradually: F: 0 → P_1



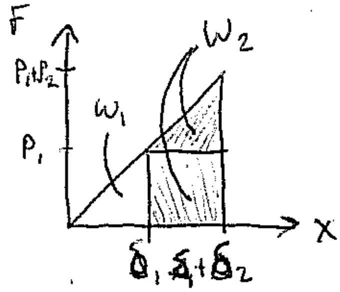


$W_1 = \frac{1}{2} P_1 \delta_1$

③ Now, Add MORE LOAD: F: $P_1 \rightarrow (P_1 + P_2)$



TOTAL WORK



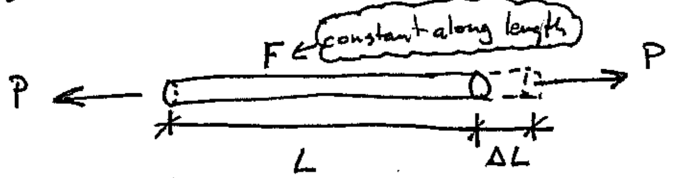
$\Sigma W = W_1 + W_2$

$W_1 = \frac{1}{2} P_1 \delta_1$

$W_2 = \frac{1}{2} P_2 \delta_2 + P_1 \delta_2$

B. Strain Energy : (U)

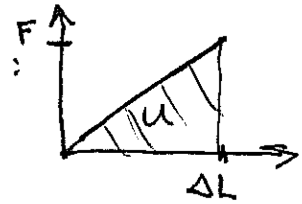
1.) Truss Bars



P = Applied load
F = Internal force
$F : 0 \to P$
L = Length
ΔL = change in length

Strain energy increases from 0 as Force is applied.

$$U = \frac{F}{2} \Delta L \longrightarrow \text{why } \frac{F}{2}? :$$



$$\epsilon = \frac{\Delta L}{L}, \quad \sigma = \frac{F}{A}$$

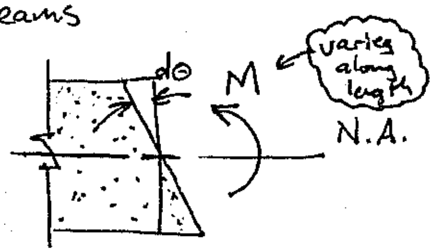
$$\sigma = E\epsilon \quad \text{(in elastic range)}$$

$$\Delta L = \frac{FL}{AE}$$

$$U = \frac{F}{2}\frac{FL}{AE} = \frac{F^2 L}{2AE}$$

$$\boxed{U = \frac{F^2 L}{2AE}}$$

2.) Beams



$$dU = \frac{M}{2} d\theta$$

$$d\theta = \frac{M}{EI} dx$$

(derivation of Moment-area Method)

$$dU = \frac{M}{2}\frac{M}{EI} dx = \frac{M^2 dx}{2EI}$$

$$\boxed{U = \int_0^L \frac{M^2}{2EI} dx}$$

C. WORK-ENERGY METHOD (Method of Real Work)

SET EXTERNAL WORK (W) EQUAL TO INTERNAL STRAIN ENERGY (U)

$$W = U$$

→ When a structure is loaded by a single force, the displacement at that force can be determined.

EXAMPLE Truss Deflection

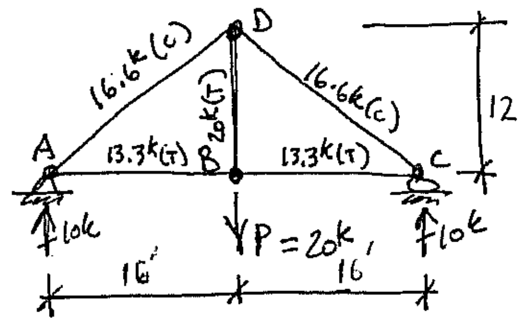
$$\frac{1}{2}P\delta = \Sigma \frac{F^2 L}{2AE}$$

Sum of Strain Energy in all bars

FIND VERTICAL DEFL. @ B

$$\delta_B = \frac{2}{P}\Sigma\frac{F^2L}{2AE}$$

E = 29,000 ksi
A = 0.5 in²



$$\delta_B = \frac{2}{20k}\left[(2)\frac{(-16.6k)^2(20ft)(12\frac{in}{ft})}{(2)(0.5\,in^2)(29000\frac{k}{in^2})} + (2)\frac{(13.3k)^2(16ft)(12\frac{in}{ft})}{(2)(0.5\,in^2)(29000\frac{k}{in^2})} + \frac{(20k)^2(12ft)(12\frac{in}{ft})}{(2)(0.5\,in^2)(29000\frac{k}{in^2})}\right]$$

$$\delta_B = \frac{2}{20}\left[4.56'' + 9.37 + 1.99''\right]$$

$$\boxed{\delta_B = 1.6''}$$

This method is extremely limited and is almost never used. Instead, consider the next lesson, method of virtual work.