Air-water exchange

• Mercury (Hg) enters lakes primarily via atmospheric deposition of particulate HgCl2 in either dry deposition or rain. In the lake, the Hg(II) is reduced to elemental mercury (Hg0) that then may volatilize from the lake surface.
• 1) Write the equation for air-water exchange of Hg.
• 2) Do you have any reason to expect the air-water exchange of Hg to be either air-phase or water-phase controlled?
• 3) What are two environmental parameters that vary seasonally that might induce seasonal changes in the mass transfer velocity for Hg evasion from a lake?
• 4) Write the equation that would allow calculation of the mass transfer coefficient based on wind speed.
III. Diffusive and dispersive processes

• A. Consider Torch Lake with its narrow constriction between the south and the central basin. Write an equation for horizontal mixing that occurs between these two basins. Be sure to define the length dimensions that are used.

• B. Rewrite the equation from part A above using two other sets of units for the dispersion coefficient. In each case, be sure to define the length parameters that are used.

• C. Write an equation for the movement of copper out of the porewaters of Torch Lake into the water column. Again, clearly define all parameters that are used in the equation.
Numerical methods

• A. Define a numerical method.
• B. For the following numerical methods, indicate the type of equation(s) with which it is used and give an example of an application of this method.
  • Gauss Elimination
  • Gauss-Seidel
  • Crude Euler
  • Heun’s Method
  • 4th order Runge-Kutta
• C. When do we use a numerical integrator instead of the analytical solution to the differential equation?
NSS

• A. What is the **general solution** to the standard mass balance equation for a substance in a lake?
• B. If the water residence time in Torch Lake is 1.1 years and the residence time for Cu is 0.5 years, how long will it take Cu to reach 95% of steady state following a one-time change in inputs?
• C. What is the analytical solution for the mass balance equation when a step change in inputs to a lake occurs? Graphically how does this solution appear?
• D. What is the analytical solution for the mass balance equation when an exponential increase in loading occurs? Graphically how does this solution appear?
Lakes In Series

• a. Consider the fate of atrazine in two lakes in series. The hydraulic residence times for the two lakes are 0.3 (first lake) and 0.7 (second lake) years. There is a pulse of the chemical into the lakes each June, and inputs are essentially zero for the remainder of the year. The primary sinks for atrazine (besides outflow) are photolysis and biodegradation; the combined rate constant for these two processes is 0.015 yr⁻¹. You wish to model the concentrations in the lakes for the 11-month period July-May.

• 1) Write the mass balance for the second of the two lakes. Use the following symbols: Q1 = flow out of lake 1; Q2 = flow out of lake 2; V1 = volume of lake 1; V2 = volume of lake 2; C1 = concentration of atrazine in lake 1; C2 = concentration in lake 2; λ1 = eigenvalue for lake 1; λ2 = eigenvalue for lake 2.

• 2) What is the eigenvalue for atrazine in each lake?

• 3) Would you apply a steady state or nonsteady state model to the lakes? Why?

• 4) What is the analytical solution (i.e., integrated form of the mass balance equation) for the second lake?

• 5) Draw a graph of the approximate shape of the concentration vs. time trend in the second lake.
VII. Feedback

• A. Consider copper in the epilimnion of Torch Lake during the summer when the lake is stratified.
• 1) Ignoring any inputs from the sediments, write the mass balance for Cu in the epilimnion considering only external loadings (rivers and precipitation), outflow, settling of particulate copper, equilibrium sorption between dissolved and particulate phases, and exchange of Cu between the epilimnion and hypolimnion.
• 2) Is there an analytical solution for the mass balance equation shown in the list of equations? If there is, write it below.
• 3) The first order rate constants for outflow, settling, and exchange across the thermocline are: a,b,c. What would be an appropriate time step to use in a numerical model of Cu concentrations in the epilimnion based only on your knowledge of the rate constants?
Equilibrium

- Many organic pollutants partition rapidly between three phases (dissolved, particulate, colloidal or DOM-bound). The partitioning is generally fast relative to other processes in the lake such that the phases are considered to be in equilibrium. As a general rule, only the particulate phase is subject to settling, only the dissolved phases is subject to air-water exchange, and only the dissolved phases is subject to chemical and biological transformation. Write the mass balance equation for an organic substance (A) in a well-mixed lake where the only sinks for compound A are settling, photolysis, and flushing. Lump all inputs of A into one source term, W. Use the notation fp = fraction particulate, fd = fraction dissolved, and fc = fraction colloidal.

- For a compound with a partition coefficient of $10^4$ L/kg in a lake with a suspended solids concentration of 1 mg/L ($10^{-6}$ kg/L), what would be the fraction of the compound in the particulate phase?