CE4505 Surface Water Quality Engineering
Fall 2009
Modeling Project 1, Part 1, Due 9/24/09

Background

Lake Alice is a eutrophic to hypereutrophic lake located in Fergus Falls, MN. The surface area of the lake is 15.9 ha, and its mean depth is 1.5 m. The lake is situated in a well-maintained, historic residential area of the town. Blooms of cyanobacteria are common, and chlorophyll concentrations commonly exceed 50 µg/L. Lake Alice's designated use is contact recreation, but this beneficial use is currently impaired by nuisance algal growth and poor transparency.

Lake Alice has no natural inlets or outlets. Since the 1930s, surface runoff from the lake's 59 ha watershed has been captured and delivered to the lake through 13 stormwater discharges. An artificial outlet was constructed with discharge to the nearby Otter Tail River. Annual precipitation in Fergus Falls is 70 cm/yr, and annual evaporation from the lake is estimated to be 78.7 cm/yr. The water outflow from the lake is 3.32×10^5 m^3/yr or 209 cm/yr.

Restoration measures under consideration include storm water diversion, sediment dredging to reduce internal nutrient loads, and flow augmentation to provide dilution and flushing. Two water sources are being considered for the flow augmentation: 1) a well located adjacent to the lake, and 2) two nearby lakes that are causing flooding of adjacent roadways.

Figure 1. Location of Fergus Falls in Minnesota, and areal view of Lake Alice.

The mass balance for phosphorus (P) states that any change in mass of P within the lake must equal the difference between inputs and the sum of outputs plus reactions:

$$V\frac{dP}{dt}=\sum inputs-\sum outputs\pm reaction$$

Inputs include atmospheric deposition (AD_P), stormwater runoff (L_P), and internal recycling from the sediments (R_P) (Fig. 2). The total input from stormwater runoff is 97.2 kg/yr or 0.61 g/m^2yr. Atmospheric deposition is not known for this location, but likely is in the range of 0.1-0.6 g/m^2yr. We shall assume a value of 0.5 g/m^2yr. The settling and recycling rates are not known separately, but the difference between the two is termed burial. The burial flux is often described mathematically as:

$$B=v_s\cdot P\cdot A$$

where the burial flux (B, g/yr) equals the settling velocity (v_s, typically in the range of 10-50 m/yr) times the lake concentration (P, g/m^3) times the lake area (A, m^2). Annual outflow (O_P) is 48 kg/yr and may be calculated as the product of concentration (P) and flow (Q, $3.32\text{x}10^5$ m^3/yr). Given the value for outflow, burial equals (49.4 + AD_P kg/yr or $AD_P + L_P - O_P$). The complete mass balance equation may be written as:

$$V\frac{dP}{dt}=AD_P+L_P-v_sP\cdot A-Q\cdot P$$

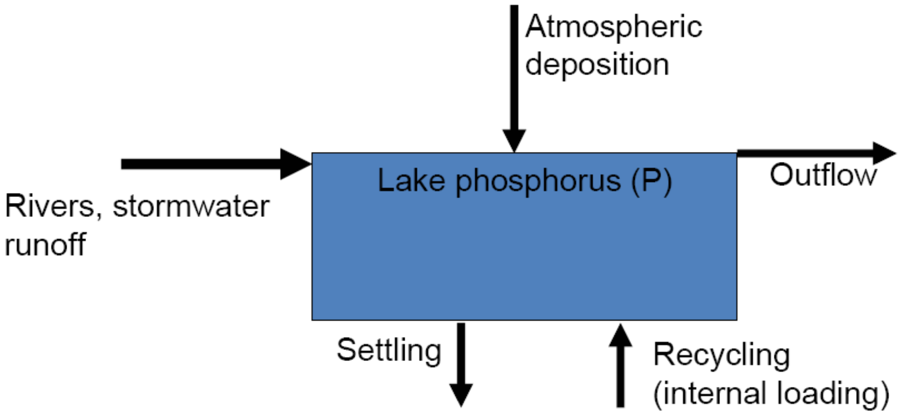

Figure 2. Schematic of the phosphorus cycle in a lake.

Assigned Tasks: For the questions below, you must explain and show how you arrive at the answer. You are to submit the project electronically including both a spreadsheet with the calculations and a word file with the text and explanations.

1. Using the mean annual fluxes given above, what is the steady state concentration of phosphorus in the lake? (0.144 g/m^3) The target concentration for a mesotrophic lake is 0.015 g/m^3.
2. What is the magnitude of the settling velocity (5.6 m/yr). What are possible reasons for the low value of v_s relative to typical values?
3. What is the hydraulic residence time in the lake?

4. What is the phosphorus residence time in the lake? Why is it much shorter than the water residence time?
5. The phosphorus loads from each of the storm water drains are listed in the table below. What would be the phosphorus concentration in the lake if each P load were removed singly? Remember that diverting a storm drain input will also change the outflow rate. To figure the new outflow rate, assume that the rate of evaporation is unchanged by the diversion.

Storm drain	P conc. (mg/m^3)	Flow (m^3/yr)	P load (g/yr)	Lake P Conc. (mg/m^3)
1	336	9651	3,240	
2	368	2825	1,040	
3	159	15065	2,400	
4	452	27306	12,300	
5	287	25187	7,230	
6	280	131821	36,900	
7	253	102632	26,000	
8	198	6238	1,240	
9	198	6238	1,240	
10	346	12476	4,320	
11	275	3296	910	
12	116	2119	250	
13	126	942	120	

6. To evaluate flow augmentation, let's assume that the water source has a P concentration of 5 mg/m^3. What flow rate of this water source would be required to reduce the P concentration in Alice Lake to 20 mg/m^3? Again, assume that the evaporation rate remains constant. Is this solution reasonable? For reference, the mean annual flow in the Otter Tail River is $3.6\text{x}10^8$ m^3/yr.
7. Dredging of sediments would reduce the internal loading of P from the sediments. Because the settling rate of P would remain the same, the fraction of settling P that is buried is increased. Mathematically, the effect is to increase the value of v_s, the settling velocity. For the settling velocities in the table below, calculate the resultant P concentrations in the lake. Based on the results in this table, is dredging an option that is worth pursuing?

Settling velocity (m/yr)	Lake P conc. (mg/m^3)
5.6	144
10	
20	
30	
50	

8. Based on your results, which option or combination of options would you recommend. Explain your answer.

Modeling Project I Part 2
Spreadsheet exercise

The analyses in Part I were sufficient to identify which remediation options are worth pursuing further (dredging), and which are not (flow augmentation, diversion). Before implementing any remediation, it is desirable to model the process in a manner that will give us more detailed predictions as to outcomes. Specifically, it would be nice to know if algal blooms are still likely to occur in summer months. In the steps below, you will model the month on a monthly basis accounting for changes in rates of precipitation and evaporation and the resultant changes in loadings of P. You will not consider seasonal changes in biological process rates, and nor will you consider the effects of lake stratification. (You will get to work with those changes in Modeling Project II.)

I. Hydrology

You are to construct a monthly hydrologic budget for the lake. The information that you have available to you is monthly precipitation amounts and monthly rates of evaporation. Enter the data below into an Excel spreadsheet to calculate the missing values in the table. You will note that the table below indicates that only a fraction of winter precipitation (January-March) is delivered to the lake; the rest is held as snow or ice on the lake until April when it all melts. Similarly, runoff in winter is only 15% of precipitation in winter and 85% of precipitation in summer. In April, the snow on the catchment (85% of winter precipitation) is allowed to melt, and 85% of this snowmelt runs into the lake.

Flow out of the artificial outlet on the lake varies with the water level in the lake according to the rating equation:

$$Q\left(\frac{m^3}{s}\right) = a\left(H - e\right)^b$$

Where a is a constant (0.009), H is the lake mean depth (m), e is an offset (0.4 m), and b is a constant (1.1). You should solve for change in storage as the sum of inputs less the sum of outputs:

$$\Delta S = P + R - O - E$$

where P is precipitation, R is runoff, O is outflow and E is evaporation. We will make the simplifying assumption that the lake surface area does not vary with lake mean depth; the error caused by this assumption should be small as long as the change in lake depth is small.

Once you have filled in all of the columns to change in lake storage, you can either calculate the next month's lake volume or mean depth. For example,

$$\text{Lake Volume}_{\text{February}} = \text{Lake Volume}_{\text{January}} + \text{Change in Storage}_{\text{January}}$$

$$H = V/A$$

Complete the table below, then answer the questions that follow.

Month	Pptn	Direct rainfal to lakel	Runoff	Outflow	Outflow	Evaporation		Change in Storage	Lake Volume	Lake mean depth (m)
	(cm)	m3/mo	m3/mo	m3/s	m3/mo	cm	m3/mo	m3/mo	m3	H (m)
Jan	7.8	1.24E+03	6.92E+03	0.008015106	2.11E+04	1.6	2.54E+03	-1.54E+04	2.07E+05	1.3
Feb	1.8	2.83E+02	1.57E+03			1.6				
Mar	5.6	8.93E+02	4.97E+03			1.6				
Apr	7.5	3.37E+04	1.00E+05			4.59				
May	4.0	6.30E+03	1.94E+04			6.732				
Jun	16.0					8.415				
Jul	6.3					10.71				
Aug	7.7					11.0925				
Sep	5.3					9.639				
Oct	4.3					8.721				
Nov	3.0					7.497				
Dec	0.8					6.579				
Sum	70.1									

1. Is the evaporation rate that you are given reasonable based on information presented to you in lecture? Explain your answer. Is it necessary to include evaporation in the water budget for this lake? What characteristics of the lake lead evaporation to be important or unimportant in this lake?
2. Comment on the simplifications made in the analysis above. What additional factors should be considered to make the analysis more realistic?

II. Nonsteady State Phosphorus Budget

You will now extend the monthly mass balance (calculated above for water) to phosphorus. Use the steady state solution to the phosphorus mass balance equation to calculate the phosphorus concentration in the pond each month. Based on this estimated concentration, calculate the monthly values for Beta, the transfer coefficient ($\beta = C/C_{in}$). Realize that C_{in} is a calculated, not a real, concentration.

a. Complete the table below. Again, complete the table in Excel and then paste into your report. For runoff, assume a constant P concentration of 281 mg/m^3. For settling, use the settling velocity calculated in Project Part 1.

Month	Direct rainfall	Runoff	Outflow	Settling	dM/dt=VdC/dt	Mass	Conc.
	g/mo	g/mo	g/mo	g/mo	g/mo	G	mg/m3
Jan	885	1,946	623	2208	0	6.11x10^3	29.6
Feb							
Mar							
Apr							
May							
Jun							
Jul							
Aug							
Sep							
Oct							
Nov							
Dec							
Sum							

1. Comment on the simplifications in the model above. What additional complexities should be considered to make the model predictions more accurate?
2. Is the average annual concentration calculated using the nonsteady state model different than that calculated in Part I for the annual steady state? Why or why not? What additional insight do you gain from the nonsteady state model.
3. To calculate the concentration in the last column of the table above, you were to apply the steady state solution ($C = W/(Q+v_sA)$) to the monthly flux values. Is it appropriate to

apply a steady-state solution for P concentration given that you are using a nonsteady state hydrological model coupled with changing monthly inputs? Why or why not?

4. What is the range in monthly values of β? What causes the monthly fluctuations. To apply your model above to the dredging remediation option, all you would have to do is to change the value of the settling velocity that is used. What is the range in monthly values in β when a settling velocity of 50 m/yr is used? What is the highest monthly P concentration that you predict with this higher settling velocity?

Modeling Project I Part 2
MatLab/Simulink Exercise

To introduce you to the MatLab/Simulink modeling environment, you are going to use template files provided to you to run a nonsteady state simulation.

A) Construct a continuous-state Simulink model for phosphorus, and verify it is working properly by using the same input conditions as in Part I and running to steady-state. Hint: Solve the mass balances you specified for phosphorus for dC/dt first.

B) Using the surface area specified in Part I and the rating curve above, construct a loop to model the volume and outflow rate of the lake over time. Lake stage height now becomes a model variable, since the water level in the lake fluctuates according to inflow and outflow rates. Run to steady-state.
Hint: Use the polynomial ("polyval") block to describe the rating curve.

C) Run your phosphorus model from (A) with the supplied loading functions and the outflow function developed in (B). Compute the annual, 30-day, and 1-day average phosphorus concentrations in the outflow. Plot the 1-day average phosphorus concentrations for a yearly cycle. How do these compare with the monthly averages calculated in Part II? How do the 30-day averages from MatLab compare with the spreadsheet (Part II). What accounts for the differences?
Hint: Run the model using a fixed timestep of 1 hour (or less), rather than the default variable timestep. This facilitates easy output averaging in Matlab.

General Matlab/Simulink tips:
Output Simulink signals to the Matlab workspace, where they can be easily manipulated and plotted. To avoid clutter in your model diagram (and resulting confusion), route signals using the "From" and "Goto" blocks. Label model components. You may also wish to experiment with subsystems. *Remember "integrator" blocks contain the initial values of state variables... these need to be specified!*