Systems connected only by feed-forward linkages: Great Lakes

Madison lakes

Mendota (Great)
Monona (Beautiful)
Wingra (Duck)
Waubesa (Swan)
Kegonsa (Fish)
Minneapolis chain of lakes
Nitrate in Great Lakes

1. What steady state concentrations are expected?

NITROGEN CYCLE

Denitrification

Inputs 5.5
Nitrification 0.02

NO$_3^-$
300 (2.5-50)

Outflow 1.4
Algal uptake 80-120
Settling 6.4
Burial 3.8

DOM inputs

Mineralization

74-116

DON 55 (15-35)

Settling 6.4 (1.5)
Mass balances, assumptions

\[ V_1 \frac{dC_1}{dt} = W_1 - Q_1 C_1 - kV_1 C_1 - v_s A_1 C_1 \]

\[ V_1 \frac{dC_2}{dt} = W_2 - Q_2 C_2 - kV_2 C_2 - v_s A_2 C_2 \]

...
\[
V_1 \frac{dC_1}{dt} = W_1 - Q_1 C_1 - kV_1 C_1 - v_s A_1 C_1 \]
\[
V_2 \frac{dC_2}{dt} = W_2 - Q_2 C_2 - kV_2 C_2 - v_s A_2 C_2 \]
\[
V_3 \frac{dC_3}{dt} = W_3 + Q_1 C_1 + Q_2 C_2 - Q_3 C_3 - kV_3 C_3 - v_s A_3 C_3 \]

\[
V_3 \frac{dC_3}{dt} = W_3 + Q_1 C_1 + Q_2 C_2 - Q_3 C_3 - kV_3 C_3 - v_s A_3 C_3 \]
\[
V_4 \frac{dC_4}{dt} = W_4 + Q_3 C_3 - Q_4 C_4 - kV_4 C_4 - v_s A_4 C_4 \]
\[
V_5 \frac{dC_5}{dt} = W_5 + Q_4 C_4 - Q_5 C_5 - kV_5 C_5 - v_s A_5 C_5 \]
### Steady state solution

\[
C_1 = \frac{W_1}{Q_1 + k_1V_1 + v_{s1}A_1}
\]

\[
C_2 = \frac{W_2}{Q_2 + k_2V_2 + v_{s2}A_2}
\]

\[
C_3 = \frac{W_3 + Q_1C_1 + Q_2C_2}{Q_3 + k_3V_3 + v_{s3}A_3}
\]

\[
C_4 = \frac{W_4 + Q_3C_3}{Q_4 + k_4V_4 + v_{s4}A_4}
\]

\[
C_5 = \frac{W_5 + Q_4C_4}{Q_5 + k_5V_5 + v_{s5}A_5}
\]

### Choice of values

<table>
<thead>
<tr>
<th>Lake</th>
<th>Superior</th>
<th>Michigan</th>
<th>Huron</th>
<th>Erie</th>
<th>Ontario</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(c) (m²)</td>
<td>1.28E+11</td>
<td>1.18E+11</td>
<td>1.35E+11</td>
<td>7.8E+10</td>
<td>6.4E+10</td>
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<tr>
<td>A (m²)</td>
<td>8.21E+10</td>
<td>5.78E+10</td>
<td>5.98E+10</td>
<td>2.52E+10</td>
<td>1.9E+10</td>
</tr>
<tr>
<td>V (m³)</td>
<td>1.2E+13</td>
<td>4.91E+12</td>
<td>3.53E+12</td>
<td>4.79E+11</td>
<td>1.63E+12</td>
</tr>
<tr>
<td>H (m)</td>
<td>146</td>
<td>85</td>
<td>59</td>
<td>19</td>
<td>86</td>
</tr>
<tr>
<td>Q (m³/yr)</td>
<td>6.7E+10</td>
<td>3.6E+10</td>
<td>1.61E+11</td>
<td>1.82E+11</td>
<td>2.12E+11</td>
</tr>
<tr>
<td>W_{ann} (Gg/yr)</td>
<td>40</td>
<td>28</td>
<td>29</td>
<td>12.2</td>
<td>9.2</td>
</tr>
<tr>
<td>W_{terr} (Gg/yr)</td>
<td>40</td>
<td>451</td>
<td>295</td>
<td>353</td>
<td>230</td>
</tr>
<tr>
<td>W (Gg/yr)</td>
<td>80</td>
<td>479</td>
<td>324</td>
<td>365</td>
<td>239</td>
</tr>
<tr>
<td>f_{forest}</td>
<td>1</td>
<td>0.25</td>
<td>0.6</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>k_{denitr.} (1/yr)</td>
<td>0.009</td>
<td>0.058</td>
<td>0.084</td>
<td>0.526</td>
<td>0.116</td>
</tr>
<tr>
<td>k_{setting} (1/yr)</td>
<td>0.002</td>
<td>0.006</td>
<td>0.006</td>
<td>0.020</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Total N loading = 1490 Gg/yr
Steady State Concentrations

Barbiero & Tuchman, 2001

FIG. 2. Box plots of selected physical and chemical variables for the Great Lakes, spring 1998. Boxes represent 25th, 50th, and 75th percentiles; whiskers indicate 10th and 90th percentiles.
Influence of L. Superior

Copper in the Keweenaw Waterway
Questions of Interest:

• Should Portage Lake Wastewater Treatment Plant be forced to control their discharge of Cu into Portage Lake?
• Does Copper originating in Torch Lake reach Lake Superior or have any effect on L. Superior?
• Is Cu in nearshore L. Superior waters coming from the Keweenaw waterway or from stamp sands in L. Superior?

PLWWTP Cu Discharge:

• Surface Water Quality Criterion (AMV) for Cu in Portage L. = 13.5 µg/L;
• Cu in effluent from WWTP averages 30 µg/L and can reach 160 µg/L;
• Cu conc. in Portage L. is about 20 µg/L;
• Does PLWWTP contribute significantly to maintaining high Cu conc. in Portage L.?
Cu Conc. In Effluent of PLWWTP

Houghton Drinking Water Plant (online 1997)
Effect of DWTP on Effluent Cu

For each lake the mass balance equation will be similar:

\[ \frac{dC}{dt} = \sum \text{Inputs} - \sum \text{Outputs} - \sum \text{Reactions} \]

The loading (W) or input will contain:

\[ W = \text{Rivers} + \text{Atmos.} + \text{upstream \cdot lakes} + \text{Po int\cdot sources} + \text{dissolution} \]

\[ W = Q_{\text{Riv.}} \cdot C_{\text{Riv.}} + P \cdot A_{\text{lake}} \cdot C_{\text{rain}} + Q_{\text{upt.}} \cdot C_{\text{upt.}} + Q_{\text{pt.source}} \cdot C_{\text{pt.source}} + v_{\text{diff}} \cdot A_{\text{source}} \cdot C_{\text{porewater}} \]
Similarly, the assimilation capacity for each lake will contain:

\[ a = Q_o + v_s A \cdot K_{sor} M + v_{diff} A_{source} \]

(biological uptake is negligible)

At Steady State, the concentration in each lake will equal

\[ C = \frac{W}{a} \]

The concentration in Torch L. will not depend on the conc. in either Portage L. or L. Superior, and the steady-state conc. in Torch L. may be calculated directly. The steady-state conc. in Portage L. will depend on the concentration in Torch L., but not on the conc. in L. Superior. Once the conc. in Torch lake has been found, the conc. in Portage L. may be calculated. With the concentration in Portage L., the concentration in L. Superior then may be calculated. The steady state conditions are shown in the following figure.
The steady state solution is obviously a simplification, but it does point out several useful facts.

- The Cu input from the wastewater treatment plant to Portage L. is miniscule compared to other inputs;

- The outflow from Torch L. is not enough to cause the measured Cu concentration in Portage L., and a major source from the sediments must exist; this sediment source must be attenuating with time;

- The atmosphere is the dominant source of Cu to L. Superior as a whole;

- The outflow from Portage L. is much larger than the estimated input of Cu to L. Superior from dissolution of stamp sands;

A time-variant model may easily be constructed using the same first order Runge-Kutta approach as used in recitation. As for the steady state model, the concentrations in Torch L. are independent of concentrations in Portage L., but for each timestep in the Portage L. model, one input is the outflow from Torch L. at that same timestep. The output from the model is shown on the following page.

\[ V \frac{dC_3}{dt} = W_3 + Q_1 C_1 + Q_2 C_2 - Q_3 C_3 - k_d C_3 V_3 - v_3 C_3 A_3 \]

For Portage L., the only new insight that we get is that the internal sedimentary source (dissolution) must be decreasing with time in Portage L.
Copper in the Keweenaw Waterway: Time-variant model

![Chart showing copper levels over time in Portage and Torch Lakes.](chart.png)