SSE 2300/CE4990: System Dynamics

Delays in Systems

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Definitions

A delay is a process where output lags behind the input in some way. The difference between the output rate and the input rate results in the accumulation of a stock in between. Think of material in transit accumulating in a stock after they arrive, as the outflow rate is less than the inflow rate. This is called a material delay. There can also be delays in a system as participants gradually adjust or change their perceptions of the world as new information becomes available. This is referred to as an information delay.

Delays can be capacity constrained or unconstrained. In capacity unconstrained delays the outflow from the stock depends only on how much material is in transit and how long its been there. The delay time is independent of the input rate and the stock in transit, and is linear. Under the assumption of unconstrained capacity, the system is effectively an uncapacitated queueing process.

The two questions to be asked of a delay are: (i) What is the average length of a delay? (ii) What is the distribution of the output around average delay (residence) time?

First Order and Higher Order Delays

A FO delay is one in which there is a single delay stock. The following equations hold for a first order delay:

$$Stock(t) = \int_{t_0}^{t} [Inflow - Outflow]dt + Stock|_{t_0} \quad (1)$$

$$Outflow = Material\ in\ transit/Delay \quad (2)$$

A higher order delays involves multiple successive delay stocks and the equations are:

$$Outflow = (Delay)_n(Inflow, Delay) \quad (3)$$

$$Total\ Material\ in\ transit = \sum_{i=1}^{n} Material\ in\ transit_i \quad (4)$$

$$Material\ in\ transit_i = \int_{t_0}^{t} Net\ Inflow_i\ dt + Material\ in\ transit_i|_{t_0} \quad (5)$$

$$Material\ in\ transit_i|_{t_0} = Inflow \times D/n \quad (6)$$

where the net inflow rate into each stock is written as:

$$Net\ Inflow_i = \begin{cases} Inflow - Exit\ rate_1, & i = 1 \\ Inflow_{i-1} - Exit\ rate_i, & i = 2, \ldots n-1 \\ Exit\ rate_{n-1} - Outflow, & i = n \end{cases} \quad (7)$$

and the exit rate from each stock, and the net outflow from the system is:

$$Exit\ rate_i = Material\ in\ transit_i/[Delay/n] \quad (8)$$

Please see the attached time rate plots to understand the equations.