SSE 2300/CE4990: System Dynamics

Linearity and order of systems

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Linear First Order System

The rate equation for a system is written in the following general form:

$$\frac{d\mathbf{S}}{dt} = f(\mathbf{S}, \mathbf{U}) \tag{1}$$

where $\mathbf{U}$ is a set of exogenous variables and $\mathbf{S}$ is a set of state variables. Each state variable represents a stock in the system. The order of a system is defined by the number of independent variables - or the number of independent stocks/state variables in the system. The system is linear if the function $f(\mathbf{S}, \mathbf{U})$ can be written as a linear combination, in the form:

$$f(\mathbf{S}, \mathbf{U}) = \mathbf{AS} + \mathbf{BU} \tag{2}$$

Where $\mathbf{A}$ and $\mathbf{B}$ are scalar matrices. Hence, for a simple linear first order system (LFOS) there is only one stock S and the function f is linear in S , as follows:

$$\frac{dS}{dt} = gS \tag{3}$$

where g is a constant. Example of a LFOS is one in which the system exhibits goal seeking behavior. The logistic model and SI models, as follows:

$$\frac{dS}{dt} = gS[1 - \frac{S}{C}] \tag{4}$$

(where C is a constant) are both examples of Non-linear first order system. The SIR model is an example of a non-linear higher order system.

The conditions for dynamic equilibrium of a system and the point of inflection respectively are:

$$\frac{dS}{dt} = 0 \tag{5}$$

$$\frac{\delta S}{\delta S} = 0 \tag{6}$$

Derive the conditions for the Logistic, SI and SIR models respectively.