The Inventory Model

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Consider the inventory problem. Inventory is stocked so that a store does not fall below demand at any given time. Problem definition is as follows:

- Replenishment of stock takes place at the end of each time period labeled $n = 0, 1, 2....$
- Total average demand during any period n is the stationary random variable ϵ_n. Hence,

$$Pr\epsilon_n = k = a_k \quad for \quad k = 0, 1, 2, ... \tag{1}$$

$$where\ a_k \geq 0 \quad and \quad \sum_{k=0}^{\infty} a_k = 1 \tag{2}$$

- A replenishment policy is established as (s, S) where $S > s$ - i.e., the stock is replenished at the end of each time period n *iff* it dips below s to the level S. The stock is checked at the ned of each time period.
- X_n denotes the stock size at the end of each time period n just before the stock is replenished (or not). $\{X_n\}$ is considered to be a stochastic process defined on the space $(-\infty, S]$.

Therefore the following relationships hold:

$$X_{n+1} = X_n - \epsilon_{n+1} \quad if\ s < X_n \leq S \tag{3}$$

$$= S - \epsilon_{n+1} \quad if\ X_n \leq s \tag{4}$$

If the successive demands $\epsilon_1, \epsilon_2, ...$ are independent random variables then the stock values $X_0, X_1, X_2...$ constitute a Markov chain whose transition probability matrix can be stated to be:

$$P_{i,j} = Pr\{X_{n+1} = j | X_n = i\} \tag{5}$$

$$= Pr\{\epsilon_{n+1} = i - j\} \qquad if\ s < i \leq S \tag{6}$$

$$= Pr\{\epsilon_{n+1} = S - j\} \qquad if\ i \leq s \tag{7}$$

Using the above formulation, consider a spare parts inventory model in which either $0, 1$ or 2 spare parts are demanded in a period with $Pr\{\epsilon_n = 0\} = 0.5, Pr\{\epsilon_n = 1\} = 0.4, Pr\{\epsilon_n = 2\} = 0.1$.