Some More Distributions

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Negative Binomial

Suppose we are required to carry out n Bernoulli trials in succession till r successes are observed ($r > 0$). Therefore, x is the number of failures or $n = r + x$. Then if the probability of a success is p, and the random variable $X =$ the number of failures that precede the rth success, then the probability of $X = x$ is given by negative binomial density function, or:

$$nb(x; r, p) = P(X = x) = P(r - 1\ S\ in\ the\ first\ x + r - 1\ trials).P(S) = C_{(r-1)}^{(x+r-1)} p^r (1-p)^x; x = 0, 1, 2, 3... \tag{1}$$

When $r = 1$ the negative binomial distribution reduces to the Geometric distribution.

Gamma Distribution

For the parameters α and β where $\alpha > 0$ and $\beta > 0$, a continuous random variable X has a Gamma distribution if the pdf of X is:

$$f(x; \alpha, \beta) = \begin{cases} \dfrac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}; x \geq 0 \\ 0;\ x < 0 \end{cases} \tag{2}$$

where $\Gamma(\alpha) = (\alpha - 1)!$ and $\Gamma(1/2) = \pi$. This function reduces tot he exponential distribution when $\lambda = 1/\beta$ and $\alpha = 1$. It can be used to characterize the time to the kth arrival in a Poisson process where $\alpha = k$. Consider writing out the equation for $k = 1$ and intuit the idea behind the distribution. However, note that in this function α can be less than 1, meaning that the function provides a model to fit different kinds of data as well.

Weibull Distribution

For the parameters α and β where $\alpha > 0$ and $\beta > 0$, a continuous random variable X has a Weibull distribution if the pdf of X is:

$$f(x; \alpha, \beta) = \begin{cases} \dfrac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}^\alpha}; x \geq 0 \\ 0;\ x < 0 \end{cases} \tag{3}$$

This function often provides a very useful fit to observed data. Note that that it reduces to an exponential distribution when $\lambda = 1/\beta$ and $\alpha = 1$. Intuitively you can think of it as a distribution for inter-arrival times in an arrival process in which the arrival rate is increasing when $\alpha > 1$ (reducing inter-arrival times) , and decreasing (increasing inter-arrival times) when $\alpha < 1$. This justifies its reduction to a Poisson when $\alpha = 1$ and the arrival rate is constant.