SSE 2300/CE 4990: Systems Design and Engineering

Review of Probability Theory

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Definitions

- **Probability:** A description of the likelihood of events - expression of uncertainty.
- **Experiment:** A process whose outcome is subject to uncertainty.
- **Sample space:** A set of possible outcomes. $S = \{1, 2, 3, 4, 5, 6\}$
- **Sample point:** A single outcome in the S. $x = \{3\}$
- **Event:** A subset of S. $E = \{1, 2, 3\}$
- $E_1, E_2, ..., E_n$ is a partition of the sample space S, if they are **mutually exclusive and collectively exhaustive**.
- **Set operations:** Union ($\cup$), Intersection ($\cap$), Complement ('), Mutually exclusive or disjoint sets, Empty set (ϕ), Collectively exhaustive.

Axioms of Probability

P(.) is a function that maps subsets of S into $[0, 1]$. The fundamental axioms are:

- For every event A, $P(A) \geq 0$.
- For the sameple space S, $P(S) = 1$
- If $\{A_i | i = 1, ..., n\}$ is a finite collection of mutually exclusive events then $P(A_1 \cup A_2 \cup ... \cup A_n) = \sum P(A_i)$

Properties that follow:

- $P(A) \leq 1$
- $\forall A, P(A) = 1 - P(A')$
- $P(\phi) = 0$
- For mutually exclusive events A and B, $P(A \cap B) = 0$
- For any two events, A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability and Bayes Theorem

$P(A|B) =$ Probability of A given that B has occurred. $P(A|B) = P(A \cap B)/P(B)$. Therefore $P(A \cap B) = P(A|B) * P(B)$.

Bayes Theorem

Let $E_1, E_2, ..., E_n$ be a partition of the sample space S with $P(E_i) > 0$ for $i = 1, ..., n$, then for any event A for which $P(A) > 0$:

$P(E_k|A) = \frac{P(A|E_k).P(E_k)}{\sum_{i=1}^{n} P(A|E_i)P(E_i)}$

Mathematical Independence

When the knowledge of event A has no impact on the probability of the event B then they are said to be mathematically independent and this relationship is expressed as:

$A \textit{ and } B \in S \textit{ are independent iff } P(B|A) = P(B), \textit{ or } P(A \cap B) = P(A).P(B)$

Law of Total Probability

Let $E_1, E_2, ..., E_n$ be a partition of the sample space S, then for any event A,

$P(A) = \sum_{i=1}^{n} P(A|E_i)P(E_i)$