Systems Design and Engineering

Problem sets

April 28, 2010

1 Classwork 1

Dated: February 13, 2010.

1.1 Problem 1

Every period that the process spends in state 0 it incurs a cost of \$2. Every period that the process spends in state 1 it incurs a cost of \$5. Every period that the process spends in state 2 it incurs a cost of \$3. What is the long run cost per period associated with this Markov chain?

1.2 Problem 2

A Markov chain $X_0, X_1, X_2...$ has the transition probability matrix:

$$\mathbf{P} = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.5 & 0.1 & 0.4 \\ 0.5 & 0.2 & 0.3 \end{bmatrix}$$

Initial Distribution: $p_0 = 0.5, p_1 = 0.5$
Calculate:

1. $P_\gamma \{X_2 = 1, X_3 = 1 | X_1 = 0\}$.
2. $P_\gamma \{X_1 = 1, X_2 = 1 | X_0 = 0\}$.

2 Classwork 2

Dated: February 22, 2010.

2.1 Problem 1

Consider a BD Process $\lambda_n = 2$ for $n = 0, 1, ...,$ and $\mu_1 = 2$ $\mu_n = 4$ for $n = 2, 3, ...$

1. Draw the rate diagram
2. Express P_n symbolically. Calculate P_0, P_1
3. What is the mean arrival rate for this system?

3 Classwork 3

Dated: March 01, 2010.

3.1 Problem 1

Consider the Markov Chain $\{X_n\}$ whose transition probability matrix is:

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ \alpha & \beta & \gamma \\ 0 & 0 & 1 \end{bmatrix}$$

Let T = $\min\{n \geq 0; X_n = 0 \text{ or } X_n = 2\}$. T is defined as the time to absorption.
Find $u = \Pr\{X_T = 0 | X_0 = 1\}$ and $v = E\{T | X_0 = 1\}$