1. A = Visa card, B = MasterCard
   \[ P(A) = 0.5, P(B) = 0.4 \]
   \[ P(A \cap B) = 0.25 \]

   (a) \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
       \[ = 0.5 + 0.4 - 0.25 \]
       \[ \Rightarrow P(A \cup B) = 0.65 \]

   (b) \[ P((A \cup B)^c) = 1 - P(A \cup B) = \frac{1}{0.35} \]

   (c) \[ P(A^c \cap B) = P(A) - P(A \cap B) = 0.5 - 0.25 = 0.25 \]

2. (a) \[ P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.25}{0.5} = \frac{0.5}{0.5} \]

   (b) \[ P(B^c \mid A) = \frac{P(B^c \cap A)}{P(A)} = \frac{0.25}{0.5} = \frac{0.5}{0.5} \]

   (c) \[ P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.25}{0.4} = \frac{0.625}{0.4} \]

   (d) \[ P(A^c \mid B) = \frac{P(A^c \cap B)}{P(B)} = \frac{0.15}{0.4} = \frac{0.375}{0.4} \]

   \[ P(A^c \cap B) = P(B) - P(A \cap B) = 0.4 - 0.25 = 0.15 \]

   (e) \[ P(A \mid A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{0.5}{0.65} = \frac{0.769}{0.65} \]

3. Suppose light aircraft disappears...
   Let \( E_1 \) = aircraft discovered, \( E_2 \) = aircraft not discovered

   \( A \) = has emergency locator; \( B \) = does not have emergency
   locator

   Given
   \[ P(E_1) = 0.70, P(E_2) = 0.30 \]
   \[ P(A \mid E_1) = 0.60 \]
   \[ P(B \mid E_2) = 0.90 \]
\[
P(A | E_1)P(E_1) = 0.7(0.6) = 0.42
\]
\[
P(A | E_2)P(E_2) = 0.1(0.3) = 0.03
\]
\[
\sum P(A | E_i)P(E_i) = 0.45
\]
\[
\frac{P(E_1 | A)P(A | E_1)}{\sum P(A | E_i)P(E_i)} = \frac{0.03}{0.45} = 0.067
\]
\[
\frac{P(E_1 | B)P(B | E_1)}{\sum P(B | E_i)P(E_i)} = \frac{0.28}{0.35} = 0.8
\]

Has locator (A)

Does not have locator (B)

Discovered (E_1)

Not discovered (E_2)

Aircraft disappears
#4.  \( P(G) = 0.25 \),  \( P(B) = 0.75 \)

\[
\text{Granite} \\
\quad \begin{array}{c}
0.25 \\
0.75 \\
\end{array}
\]

\[
\begin{array}{c}
R_1 < R_2 < R_3 \\
R_1 < R_3 < R_2 \\
R_3 < R_1 < R_2 \\
R_1 < R_2 < R_3 \\
R_1 < R_3 < R_2 \\
R_3 < R_1 < R_2 \\
\end{array}
\]

(a) \[ P(G \mid R_1 < R_2 < R_3) = \frac{P(R_1 < R_2 < R_3 \mid G) P(G)}{P(R_1 < R_2 < R_3 \mid G) P(G) + P(R_1 < R_2 < R_3 \mid B) P(B)} \]

\[ = \frac{0.60 \cdot 0.25}{0.60 \cdot 0.25 + 0.10 \cdot 0.75} = 0.67 \]

\[ \Rightarrow P(G \mid R_1 < R_2 < R_3) = 0.67 \]

\[ \therefore P(B \mid R_1 < R_2 < R_3) = 0.33 \]

\[ P(G \mid R_1 < R_2 < R_3) > P(B \mid R_1 < R_2 < R_3) \]

\[ \Rightarrow \text{Classify rock as granite given reading } R_1 < R_2 < R_3 \]
(b) \[ P(G \mid R_1 < R_3 < R_2) = \frac{P(R_1 < R_3 < R_2 \mid G) P(G)}{P(R_1 < R_3 < R_2 \mid G) P(G) + P(R_1 < R_3 < R_2 \mid B) P(B)} \]
\[ = \frac{0.25(0.25)}{0.25(0.25) + 0.20(0.75)} \]
\[ \therefore P(G \mid R_1 < R_3 < R_2) = 0.294 < P(B \mid R_1 < R_3 < R_2) \]
\[ \Rightarrow \text{Classify as basalt if reading is } R_1 < R_3 < R_2 \]

\[ P(G \mid R_3 < R_1 < R_2) = \frac{P(R_3 < R_1 < R_2 \mid G) P(G)}{P(R_3 < R_1 < R_2 \mid G) P(G) + P(R_3 < R_1 < R_2 \mid B) P(B)} \]
\[ = \frac{0.15(0.25)}{0.15(0.25) + 0.170(0.75)} \]
\[ \therefore P(G \mid R_3 < R_1 < R_2) = 0.067 < P(B \mid R_3 < R_1 < R_2) \]
\[ \Rightarrow \text{Classify as basalt if reading is } R_3 < R_1 < R_2 \]

(c) \[ p(\text{erroneous classification}) = P(B \text{ classified as } G) + P(G \text{ classified as } B) \]
\[ = P(\text{classify as } G \mid B) P(B) + P(\text{classify as } B \mid G) P(G) \]
\[ = P(R_1 < R_2 < R_3 \mid B) P(B) + P(R_1 < R_3 < R_2 \text{ or } R_3 < R_1 < R_2 \mid B) P(B) \]
\[ = 0.10(0.75) + (0.25 + 0.15)0.25 \]
\[ \Rightarrow [0.175] \]
(d) \( \Pr(G) = p \), \( \Pr(B) = 1 - p \)

\[ \Rightarrow \text{To always classify as granite, need value of } p \text{ for which:} \]
\[ \Pr(G | R_1 < R_2 < R_3) > 0.5 \]
\[ \Pr(G | R_1 < R_3 < R_2) > 0.5 \]
\[ \Pr(G | R_3 < R_1 < R_2) > 0.5 \]

\[ \Pr(G | R_1 < R_2 < R_3) = \frac{0.6p}{0.6p + 0.1(1-p)} > 0.5 \text{ iff } p > \frac{1}{7} \]

\[ \Pr(G | R_1 < R_3 < R_2) = \frac{0.25p}{0.25p + 0.2(1-p)} > 0.5 \text{ iff } p > \frac{4}{9} \]

\[ \Pr(G | R_3 < R_1 < R_2) = \frac{0.15p}{0.15p + 0.7(1-p)} > 0.5 \text{ iff } p > \frac{14}{17} \]

Most restrictive

\[ \therefore \text{will always classify as granite if } p > \frac{14}{17} \]