CE/CSE 5710: Modeling & Simulation

*Problems**

March 26, 2012

Problem 1

Consider a BD Process $\lambda_n = 2$ for $n = 0, 1, ...,$ and $\mu_1 = 2$ $\mu_n = 4$ for $n = 2, 3, ...$

1. Draw the rate diagram
2. Express P_n symbolically. Calculate P_0, P_1
3. What is the mean arrival rate for this system?

Problem 2

You are told that a small single-server, birth-death-type queue with finite capacity cannot hold more than 3 customers. The three arrival or birth rates are $(\lambda_0, \lambda_1, \lambda_2) = (3, 2, 1)$, while the service or death rates are $(\mu_1, \mu_2, \mu_2) = (1, 2, 2)$. Find the following:

- Steady-state probabilities $\{p_i, i = 0, 1, 2, 3\}$ (5)
- L - the expected number of customers in the system (3)
- The average arrival rate $\lambda_{eff} = \sum \lambda_i p_i$ (1)
- The expected system waiting time W (1)

Problem 3

An airlines reservation system has two computers, one on-line and one backup. The operating computer fails after an exponentially distributed duration having parameter μ and is replaced by the standby. Both computers cannot fail at the same time. There is one repair facility, and the repair facility times are exponentially distributed with parameter λ. Let $X(t)$ be the number of computers in operating condition at time t. Then find the following:

- Draw the Markov chain for $X(t)$, assuming that it can transition states only in a discrete fashion triggered by computers failing or being repaired. (Once a computer fails its not in operating condition any more. Once it is repaired its back in operating condition.) (1)
- Steady-state probabilities $\{P_n, n = 0, 1, 2, \}$ (2)
- The probability that at least one computer is operating. (2)

*Material in this chapter is adapted from: *F. S. Hillier and G. J. Lieberman. Introduc- tion to Operations Research. McGraw-Hill, 8th. edition, 2004.* and *H. M. Taylor and S. Karlin. An Introduction to Stochastic Modeling. Academic Press, 3rd. edition, 1998.*

Problem 4

Kolmogorov's forward equation for a continuous time Markov chain can be written in matrix form as:

$$\mathbf{p'(t)} = \mathbf{p(t)}\ \mathbf{Q}$$

where $\mathbf{p}(t)$ is the vector of the probabilities $(p_0(t), p_1(t), \cdots)$, $\mathbf{p'}(t)$ is its derivative and $\mathbf{Q}$ is the infinitesimal generator matrix. Then:

(4)

- Solve this equation given that $\mathbf{p}(0) = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. [Hint: $e^x = \sum_{n=1}^{\infty} \frac{x^n}{n!}$]

(5)

- If for a 2 state Markov chain (states 0,1):

$$\mathbf{Q} = \begin{bmatrix} -\alpha & \alpha \\ \beta & -\beta \end{bmatrix}$$

  then find $\mathbf{Q}^n$ [Hint: Express $\mathbf{Q}^n$ as a function of $\mathbf{Q}$]

(4)

- If $\pi = \frac{\alpha}{\alpha+\beta}$ and $\tau = \alpha + \beta$ then express the matrix $\mathbf{p}(t)$ in terms of π and τ.

Problem 5

For the County Hospital emergency room the management engineer has concluded that patients arrive randomly (a Poisson input process) at the rate of 1 every half hour. The time spent by a doctor treating the cases also follow an exponential distribution - on an average of 20 minutes.

Currently there is one doctor in charge. Is hiring a second doctor a good idea? Calculate the following metrics to justify:

	s=1	**s=2**
ρ		
P_0		
P_1		
P_n for $n \geq 2$		
L_q		
L		
W_q		
W		

Decision criteria:
$E(TC)$ = Expected total cost per functional unit
$E(SC)$ = Expected service cost per functional unit
$E(WC)$ = Expected waiting cost per functional unit
The objective is to choose the number of servers so as to:
Minimize $E(TC) = E(SC) + E(WC)$

Problem 6

Ms. Cutt runs a one person tax consultant firm. She does not make appointments, but runs her service on a first come, first serve basis. She finds that she is extremely busy during March and April, so she is considering hiring a part time assistant or even possibly increasing the size of her facility. Having obtained a degree in SSE from Michigan Tech in her spare time, prior to embarking upon her career, she elects to analyze the system carefully before making a decision.

She notices that on any given day during March or April, customers arrive according to a Poisson process and the mean arrival rate is 5 per hour. Because of her excellent reputation customers are usually willing to wait. The data further showed that the customer processing time is 10 minutes. Help her answer the following questions (Many years of tax law has resulted in her forgetting most of her SSE knowledge!).

- Average number of customers in the shop
- Average number of customers waiting for service
- Average time customers spend in the system and
- Average time customers spend waiting.
- Average number waiting when there is at least one person waiting
- Percentage of time that an arrival can walk right in without having to wait
- Ms. Cut's facility currently only has four seats. She wants to know the probability that a customer upon arrival will not be able to find a seat

What would you suggest?