First Step Analysis

1.) Consider the three state Markov Chain \( \{X_n\} \).

with probability transition matrix

\[
\mathbf{P} = \begin{bmatrix}
0 & 1 & 2 \\
1 & \beta & \gamma \\
2 & 0 & 0 \\
\end{bmatrix}
\]

\( \beta > 0 \)
\( \gamma > 0 \)

Calculate the following:

(i) \( u = \Pr \{ X_T = 0 \mid X_0 = 1 \} \).

(ii) \( v = E \{ T \mid X_0 = 1 \} \).

Note it is expected that the MC will be absorbed in state 0, given that \( X_0 = 1 \) in \( T \) steps.

2.) For a Markov Chain \( \{X_n\} \) with probability transition matrix as follows:

\[
\mathbf{P} = \begin{bmatrix}
0 & 1 & 2 & 3 \\
0 & 0.4 & 0.3 & 0.2 & 0.1 \\
1 & 0.1 & 0.3 & 0.3 & 0.3 \\
2 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

- Find the probability that the MC gets absorbed in state 0, given that it begins in state \( X_0 = 1 \).
- Mean absorption time.