CE 5710 - Modeling & Simulation

Homework 2: Due February 13, 2012

Arguments derived from probabilities are idle

~ Plato

February 2, 2012

Problem 1

Derive the Poisson process by using the assumption that the number of arrivals in non-overlapping intervals are statistically independent and then applying the binomial distribution.
[Hint: Consider the following approximation:

$$[1 - \frac{\lambda}{n}]^n \simeq e^{\lambda} \tag{1}$$

for very large values of n and moderate values of λ.]

Problem 2

You are given two Poisson processes with intensities λ_1 and λ_2. Find the probability that there is an occurrence of the first stream before the second, starting at time $t = 0$.
[Hint: Use the memoryless property].

Problem 3

(i) A Markov chain $X_0, X_1, X_2...$ has the transition probability matrix:

$$\mathbf{P} = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

Calculate:

1. $P_\gamma \{X_2 = 1, X_3 = 1 | X_1 = 0\}$.
2. $P_\gamma \{X_1 = 1, X_2 = 1 | X_0 = 0\}$.

(ii) A Markov chain $X_0, X_1, X_2...$ has the transition probability matrix:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.2 & 0.2 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix}$$

Calculate:

1. The 2-step transition matrix P^2
2. $P_\gamma \{X_3 = 1 | X_1 = 0\}$.
3. $P_\gamma \{X_3 = 1 | X_0 = 0\}$.

Problem 4

A Markov chain $X_0, X_1, X_2...$ has the transition probability matrix:

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.1 & 0.4 & 0.1 & 0.4 \\ 0.2 & 0.1 & 0.6 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Use First Step Analysis to calculate

- The probability of absorption in state 0 given that the initial state X_0 is 1 and 2 respectively.
- The expected time to absorption in state 0 given that the initial state X_0 is 1 and 2 respectively.

Problem 5

Consider a directed graph G, consisting of 6 vertices: $V = \{v_1, ..., v_6\}$ where each of the nodes $v_1, ...v_5$ are positioned on the vertices of a pentagon and the node v_6 is positioned in its centroid. The set of edges are:
$E = \{e_{2,6}, e_{3,6}, e_{2,3}, e_{1,2}, e_{3,4}, e_{1,6}, e_{4,6}, e_{6,2}, e_{6,3}, e_{3,2}, e_{2,1}, e_{4,3}, e_{6,1}, e_{6,4}, e_{6,5}, e_{4,6}, e_{1,6}\}$
If the node 5 is an absorbing node, then estimate the expected number of steps that will be necessary for a random walker to land up at node 5 given that he may start at any of the other possible nodes.

Problem 6

An airline system has 2 computers only one of which is in operation at any given time. A computer can break down on any given day with probability p. There is a single repair facility that takes 2 days to repair the machine. Only one computer at a time ca be repaired and both machines are extremely unlikely to fail on the same day. Form a Markov chain by taking as states the pairs (x, y) where x is the number of machines in operation at the end of the day, and $y = 1$ if a day's work has been expended on a machine not yet repaired and 0 otherwise. Calculate the reliability of the system defined by the percentage of the time that at least one computer is functioning.