Problem 1

A Markov Chain has the transition probability matrix:

\[ \mathbf{P} = \begin{bmatrix}
0 & 1 & 2 \\
0 & 0.7 & 0.2 & 0.1 \\
1 & 0 & 0.6 & 0.4 \\
2 & 0.5 & 0 & 0.5 \\
\end{bmatrix} \]

Calculate:

(i) \( \Pr \{ X_2 = 1, X_3 = 1 | X_1 = 0 \} \).

(ii) \( \Pr \{ X_1 = 1, X_2 = 1 | X_0 = 0 \} \).

Problem 2

A Markov Chain \( X_0, X_1, X_2 \ldots \) has the transition probability matrix:

\[ \mathbf{P} = \begin{bmatrix}
0 & 1 & 2 \\
0 & 0.3 & 0.2 & 0.5 \\
1 & 0.5 & 0.1 & 0.4 \\
2 & 0.5 & 0.2 & 0.3 \\
\end{bmatrix} \]

Initial Distribution: \( \mathbf{\pi}_0 = 0.5; \mathbf{\pi}_1 = 0.5 \).

Calculate:

(i) \( \Pr \{ X_0 = 1, X_1 = 1, X_2 = 0 \} \).

(ii) \( \Pr \{ X_1 = 1, X_2 = 1, X_3 = 0 \} \).
A Markov Chain $X_n$ has states $0, 1, 2$

Transition probability matrix

$$P^2 = \begin{bmatrix}
0 & 0 & 1 & 2 \\
0 & 0.1 & 0.2 & 0.7 \\
1 & 0.2 & 0.2 & 0.6 \\
2 & 0.6 & 0.1 & 0.3
\end{bmatrix}$$

Calculate:

(i) The 2-step transition matrix $P^2$.

(ii) $P \{ X_3 = 1 \mid X_1 = 0 \}$.

(iii) $P \{ X_3 = 1 \mid X_0 = 0 \}$. 