The Arrival Process

Assumptions

We will derive the probability distribution function that governs the arrival process and show that it results in the inter-arrival times being governed by the exponential distribution function. The arrival counting process is given by \( \{N(t), t \geq 0\} \) where \( N(t) \) denotes the total number of arrivals up to time \( t \) and \( N(0) = 0 \). In addition the following three assumptions will need to be satisfied.

- Probability that an arrival occurs between \( t \) and \( (t + \Delta t) \) is given by \( \lambda \Delta t + o(\Delta t) \) where \( \Delta t \) is an incremental element and the value of \( o(\Delta t) \) compared to the value of \( \Delta t \) is negligible as \( \Delta t \) tends to 0:
  \[
  \lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0 \tag{1}
  \]

- Probability of more than 1 arrival between \( t \) and \( t + \Delta t = o(\Delta t) \)

- Number of arrivals in non-overlapping intervals are statistically independent

We wish to calculate \( p_n(t) \) the probability of \( n \) arrivals in a time interval of length \( t \), where \( n \) is an integer \( \geq 0 \).