#1. A = Visa card; B = MasterCard

$P(A) = 0.5$, $P(B) = 0.4$

$P(A \cap B) = 0.25$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.4 - 0.25$

$\Rightarrow \boxed{P(A \cup B) = 0.65}$

(b) $P((A \cup B)^c) = 1 - P(A \cup B) = \boxed{0.35}$

(c) $P(A \cap B^c) = P(A) - P(A \cap B) = 0.5 - 0.25 = \boxed{0.25}$

#2. (a) $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.25}{0.5} = \boxed{0.5}$

(b) $P(B^c|A) = \dfrac{P(B^c \cap A)}{P(A)} = \dfrac{0.25}{0.5} = \boxed{0.5}$

(c) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.25}{0.4} = \boxed{0.625}$

(d) $P(A^c|B) = \dfrac{P(A^c \cap B)}{P(B)} = \dfrac{0.15}{0.4} = \boxed{0.375}$

$P(A^c \cap B) = P(B) - P(A \cap B) = 0.4 - 0.25 = 0.15$

(e) $P(A | A \cup B) = \dfrac{P(A \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(A)}{P(A \cup B)} = \dfrac{0.5}{0.65} = \boxed{0.769}$

#3. Suppose light aircraft disappears...

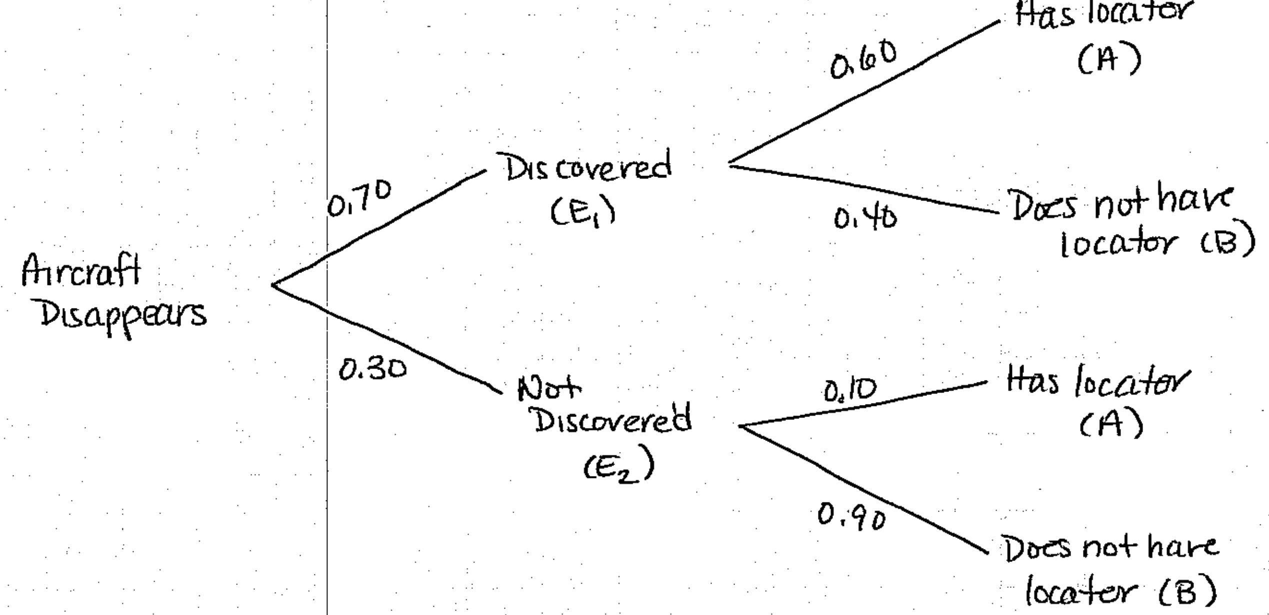
Let E_1 = aircraft discovered; E_2 = aircraft not discovered

A = has emergency locator; B = does not have emergency locator

Given $P(E_1) = 0.70$, $P(E_2) = 0.30$

$P(A|E_1) = 0.60$

$P(B|E_2) = 0.90$



$P(A|E_1)P(E_1) = 0.7(0.6) = 0.42$

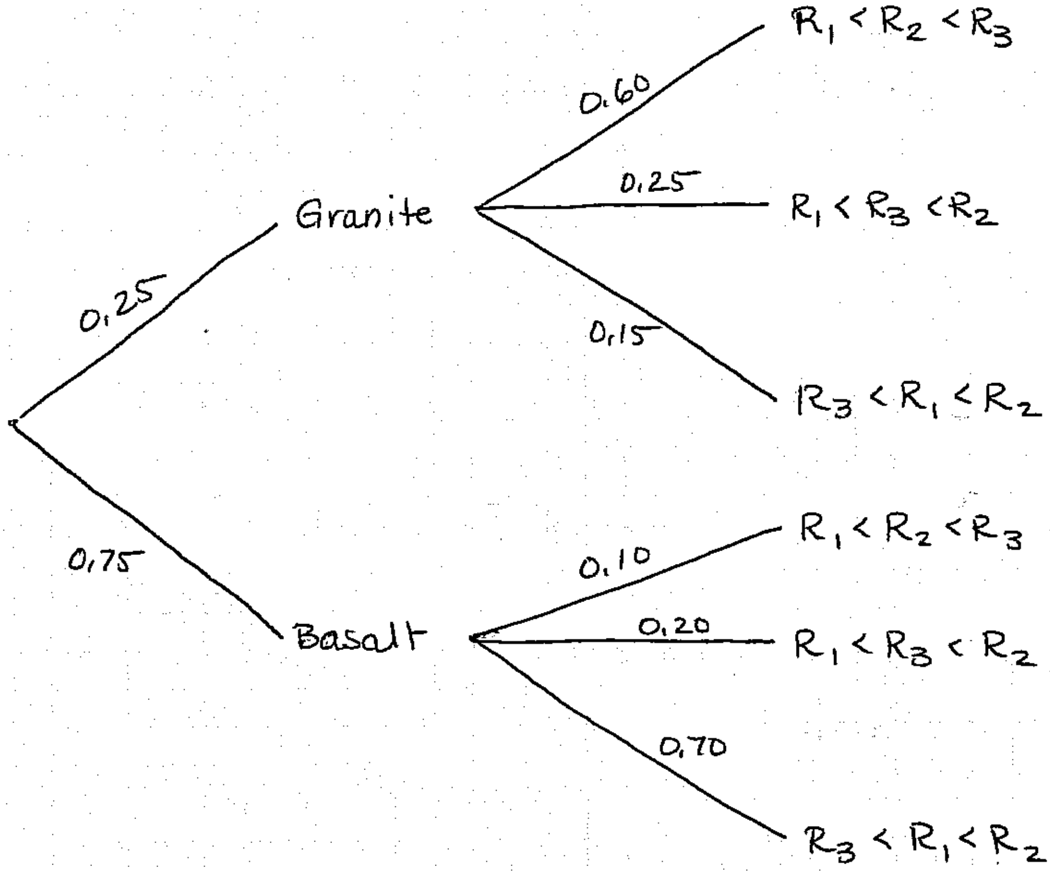
$P(A|E_2)P(E_2) = 0.10(0.30) = 0.03$

$\sum P(A|E_i)P(E_i) = 0.45$

(a) $P(E_2|A) = \dfrac{P(A|E_2)P(E_2)}{\sum P(A|E_i)P(E_i)} = \dfrac{0.03}{0.45} = \boxed{0.067}$

(b) $P(E_1|B) = \dfrac{P(B|E_1)P(E_1)}{\sum P(B|E_i)P(E_i)} = \dfrac{0.4(0.7)}{0.4(0.7) + 0.9(0.3)} = \dfrac{0.28}{0.55} = \boxed{0.509}$

#4. $P(G) = 0.25$, $P(B) = 0.75$



(a) $$P(G \mid R_1 < R_2 < R_3) = \frac{P(R_1<R_2<R_3 \mid G)\,P(G)}{P(R_1<R_2<R_3 \mid G)\,P(G) + P(R_1<R_2<R_3 \mid B)\,P(B)}$$

$$= \frac{0.60(0.25)}{0.60(0.25) + 0.10(0.75)} = 0.67$$

$\Rightarrow P(G \mid R_1 < R_2 < R_3) = 0.67$

$\therefore P(B \mid R_1 < R_2 < R_3) = 0.33$

$P(G \mid R_1 < R_2 < R_3) > P(B \mid R_1 < R_2 < R_3)$

↳ Classify rock as granite given reading $R_1 < R_2 < R_3$

(b) $$P(G \mid R_1 < R_3 < R_2) = \frac{P(R_1<R_3<R_2 \mid G)P(G)}{P(R_1<R_3<R_2 \mid G)P(G) + P(R_1<R_3<R_2 \mid B)P(B)}$$

$$= \frac{0.25(0.25)}{0.25(0.25) + 0.20(0.75)}$$

$$\therefore P(G \mid R_1<R_3<R_2) = 0.294 < P(B \mid R_1<R_3<R_2)$$

$\Rightarrow$ Classify as basalt if reading is $R_1 < R_3 < R_2$

$$P(G \mid R_3<R_1<R_2) = \frac{P(R_3<R_1<R_2 \mid G)P(G)}{P(R_3<R_1<R_2 \mid G)P(G) + P(R_3<R_1<R_2 \mid B)P(B)}$$

$$= \frac{0.15(0.25)}{0.15(0.25) + 0.70(0.75)}$$

$$\therefore P(G \mid R_3<R_1<R_2) = 0.067 < P(B \mid R_3<R_1<R_2)$$

$\Rightarrow$ Classify as basalt if reading is $R_3 < R_1 < R_2$

(c) P(erroneous classification) $= P(B \text{ classified as } G) + P(G \text{ classified as } B)$

$$= P(\text{classify as } G / B)P(B) + P(\text{classify as } B/G)P(G)$$

$$= P(R_1<R_2<R_3 \mid B)P(B) + P(R_1<R_3<R_2 \text{ or } R_3<R_1<R_2 \mid G)P(G)$$

$$= 0.10(0.75) + (0.25+0.15)0.25$$

$$\Rightarrow \boxed{0.175}$$

(d) $P(G) = p$, $P(B) = 1-p$

↳ To always classify as granite, need value of p for which:

$$P(G \mid R_1 < R_2 < R_3) > 0.5$$
$$P(G \mid R_1 < R_3 < R_2) > 0.5$$
$$P(G \mid R_3 < R_1 < R_2) > 0.5$$

$$P(G \mid R_1 < R_2 < R_3) = \frac{0.6p}{0.6p + 0.1(1-p)} > 0.5 \text{ iff } p > 1/7$$

$$P(G \mid R_1 < R_3 < R_2) = \frac{0.25p}{0.25p + 0.2(1-p)} > 0.5 \text{ iff } p > 4/9$$

$$P(G \mid R_3 < R_1 < R_2) = \frac{0.15p}{0.15p + 0.7(1-p)} > 0.5 \text{ iff } p > 14/17$$

↑ Most restrictive

∴ Will always classify as granite if $p > 14/17$