

# CEE 5690 - Descriptive Modeling of Data

## Homework 2

*We are caught in an inescapable network of mutuality,  
tied in a single garment of destiny. Whatever affects one directly,  
affects all indirectly.*

Martin Luther King, Jr.

January 28, 2008

### Question 1

Let  $G$  be a graph of  $n$  nodes, then prove that any two of the following statements imply the third:

- (i)  $G$  is connected
- (ii)  $G$  does not contain a cycle
- (iii)  $G$  has  $n - 1$  edges

Also show that proving the above is equivalent to stating *Every  $n$ -node tree has exactly  $n-1$  edges.*

### Question 2

Devise an algorithm to test if a given graph is bipartite.

### Question 3

Some friends of yours work on wireless networks, and they are currently studying the properties of a network of  $n$  mobile devices (let us assume  $n$  is even). As the devices move around (or as your friends move around!) they define a graph at any point in time as follows: there is a node representing each of the  $n$  devices, and there is an edge between device  $i$  and device  $j$  if the physical locations of  $i$  and  $j$  are no more than 500m apart. If so, we say that  $i$  and  $j$  are “in range” of each other.

Help your friend by defining a constraint that will guarantee that the network always remains connected. Will it be enough to make sure that at any point of time every node has a degree of at least  $\frac{n}{2}$ .